Short Labeling Scheme for Connectivity Check on Certain Graph Classes of Unbounded Clique-Width

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Let G be a planar graph with n vertices. It is proved in [1] that we can assign to each vertex x of G a bit sequence $J(x)$ of size at most $O(\log(n))$ such that for every pair of vertices (x, y) and every subset X of $V(G) - \{x, y\}$, we can verify if x and y are connected in $G \backslash X$ just by looking at $J(x), J(y)$ and $\{J(z) \mid z \in X\}$. A similar result is known for graph classes of bounded clique-width [2]. In order to extend this result to more graph classes we investigate graph classes obtained by "gluing" graphs of bounded clique-width with limited overlaps. Our objective is to combine the two labelings. For that we introduce a notion of decomposition that extends the one of *tree-decomposition*.

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two graph classes. An $(\mathcal{H}_1, \mathcal{H}_2)$-*decomposition* of a graph $G = (V(G), E(G))$ is a partition $\mathcal{T}$ of $E(G)$ such that:

1. for every U in $\mathcal{T}$ the sub-graph $G[U]$ is in $\mathcal{H}_2$,
2. the intersection graph[1] $G(\mathcal{T})$ of $\mathcal{T}$ is in $\mathcal{H}_1$.

The *spread* of an $(\mathcal{H}_1, \mathcal{H}_2)$-decomposition is $\max_{x \in V_G} \Big| \{E \in \mathcal{T} \mid x \text{ incident with an edge of } E\} \Big|$. The $(\mathcal{H}_1, \mathcal{H}_2)$-*width* of an $(\mathcal{H}_1, \mathcal{H}_2)$-decomposition $\mathcal{T}$ is the maximum between its spread and the maximum degree of its intersection graph $G(\mathcal{T})$. The $(\mathcal{H}_1, \mathcal{H}_2)$-*width* of a graph G is the minimum over all $(\mathcal{H}_1, \mathcal{H}_2)$-decompositions.

We let $\mathcal{P}$ be the class of planar graphs and $CWD(\leq k)$ be the class of graphs of clique-width at most k. We prove that if G has a $(\mathcal{P}, CWD(\leq k))$-decomposition of $(\mathcal{P}, CWD(\leq k))$-width ℓ then we can assign to each vertex x of G a bit sequence $J(x)$ of size at most $O(f(k, \ell) \cdot \log(n))$ such that for every pair of vertices (x, y) and every subset X of $V(G) - \{x, y\}$, we can verify if x and y are connected in $G \backslash X$ just by looking at $J(x), J(y)$ and $\{J(z) \mid z \in X\}$. For instance $K_{3,3}$-minor free graphs of bounded degree have a $(\mathcal{P}, CWD(\leq 3))$-decomposition of bounded $(\mathcal{P}, CWD(\leq 3))$-width. This result is available in [3].

References

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[2] B. Courcelle and R; Vanicat. Query Efficient Implementation of Graphs of Bounded Clique-Width. *Discrete Applied Mathematics*, 131:129-150, 2003.

[3] M.M. Kanté. *Graph Structurings: Some Algorithmic Applications.* PhD Thesis, Université Bordeaux 1, 2008. Available in http://www.labri.fr/perso/kante/research.php.

[1] An intersection graph of a cover $\mathcal{T}$ is an undirected graph $G(\mathcal{T})$ with vertex set $\{x_U \mid U \in \mathcal{T}\}$ and edge set $\{x_U x_V \mid U \cap V \neq \emptyset\}$