Coloration, ensemble indépendant et structure de graphe

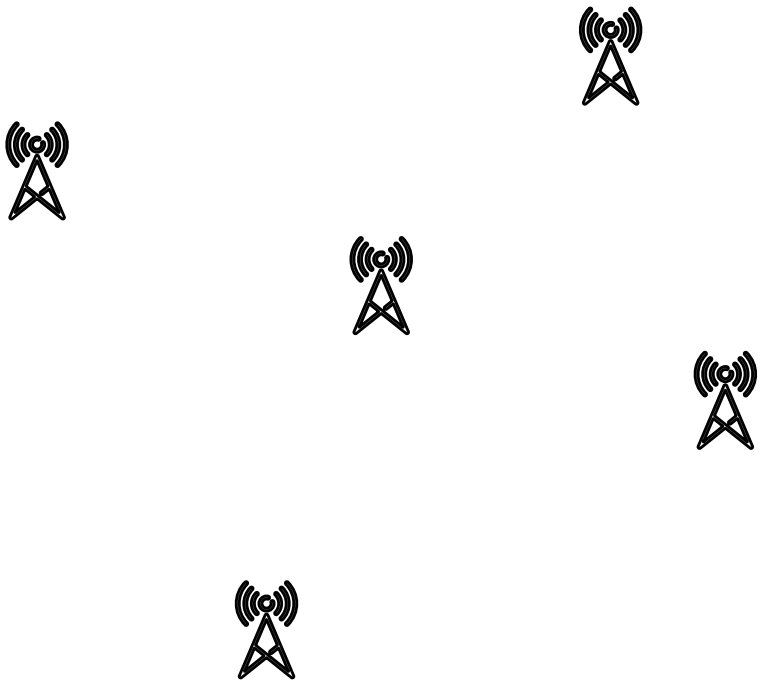
Soutenance de thèse

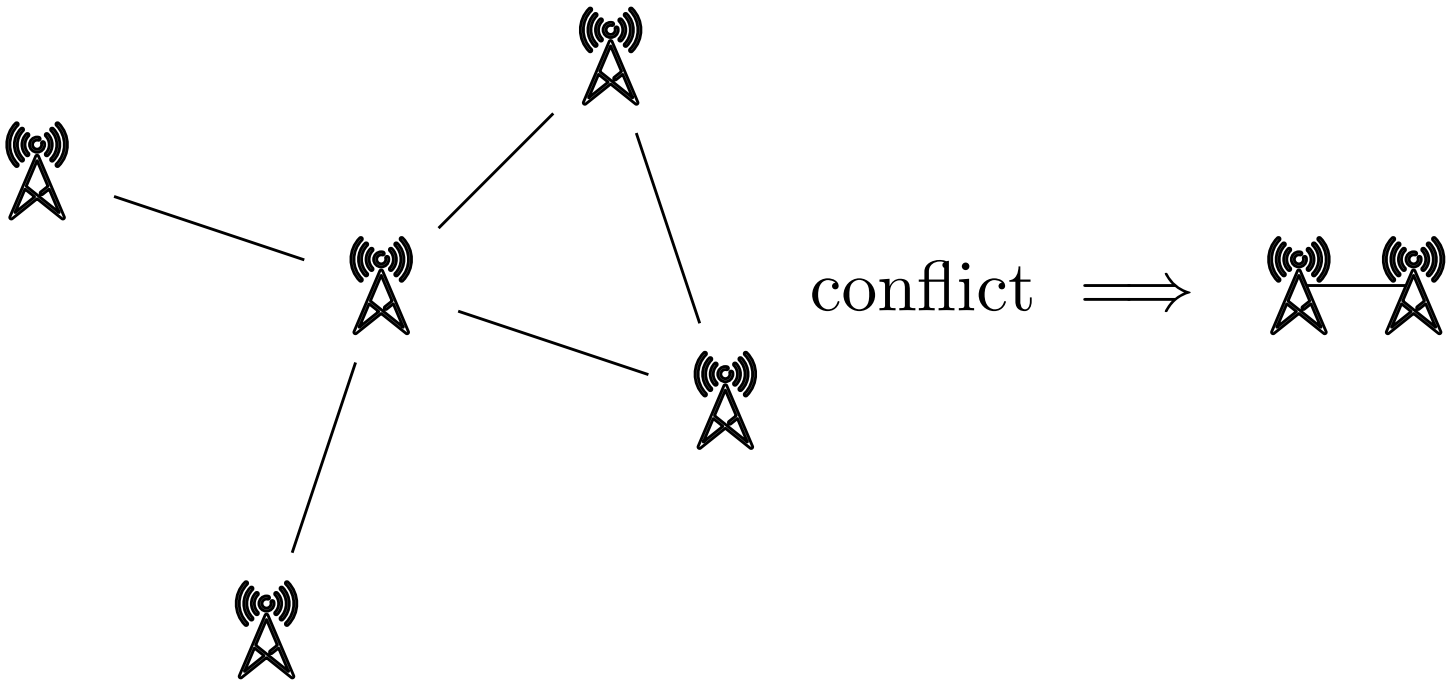
Lucas Pastor

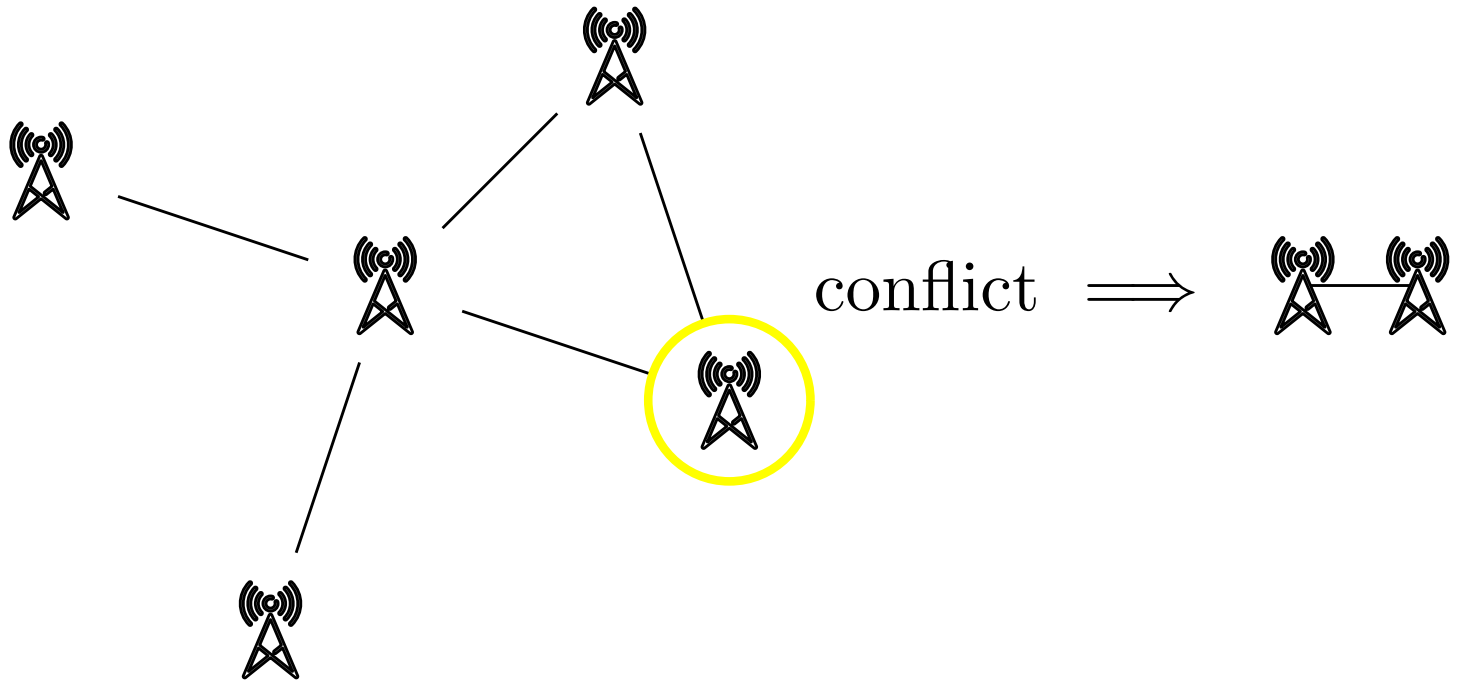
Rapporteurs : Frédéric Havet et Dieter Rautenbach.
Examinateurs·trices : Laurent Beaudou, Mickael Montassier, Aline Parreau et Nicolas Trotignon.
Directeurs : Sylvain Gravier et Frédéric Maffray.

Laboratoire G-SCOP

Jeudi 23 novembre 2017




conflict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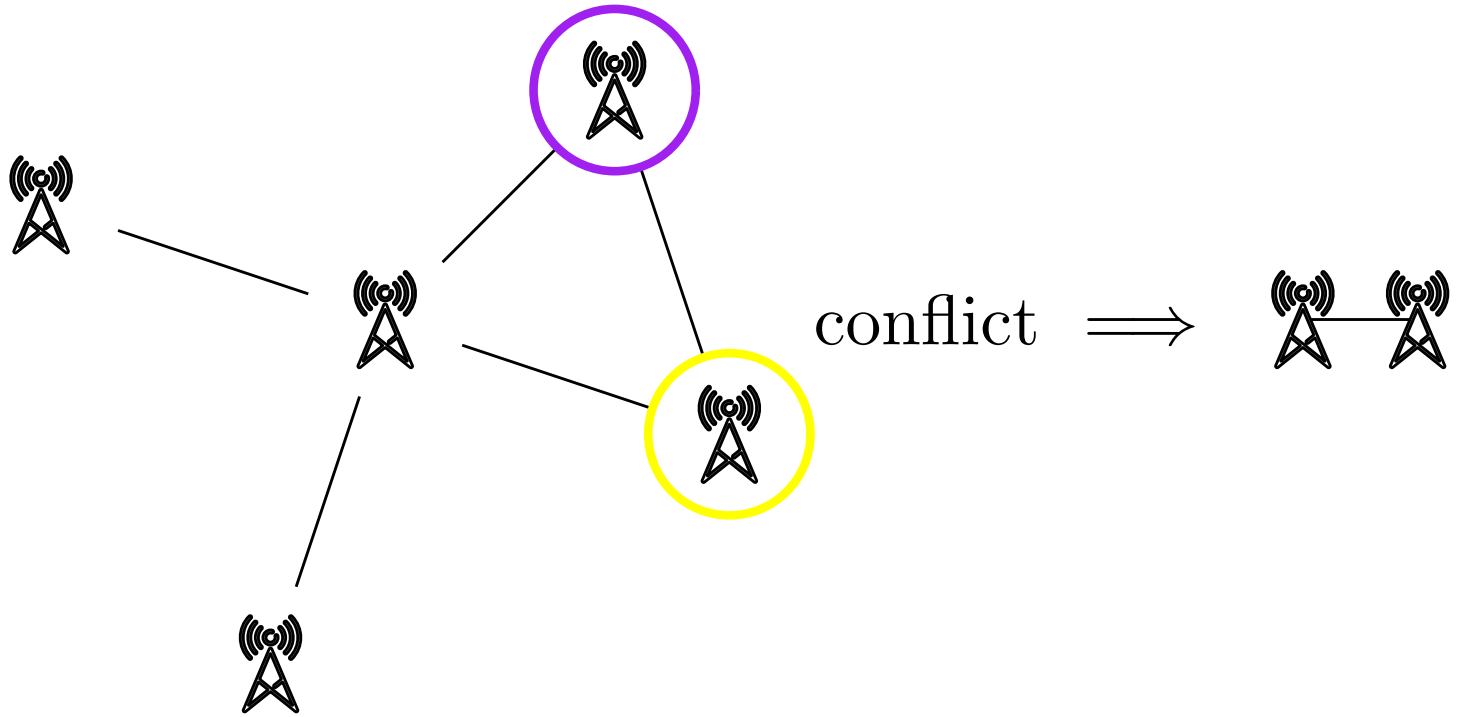
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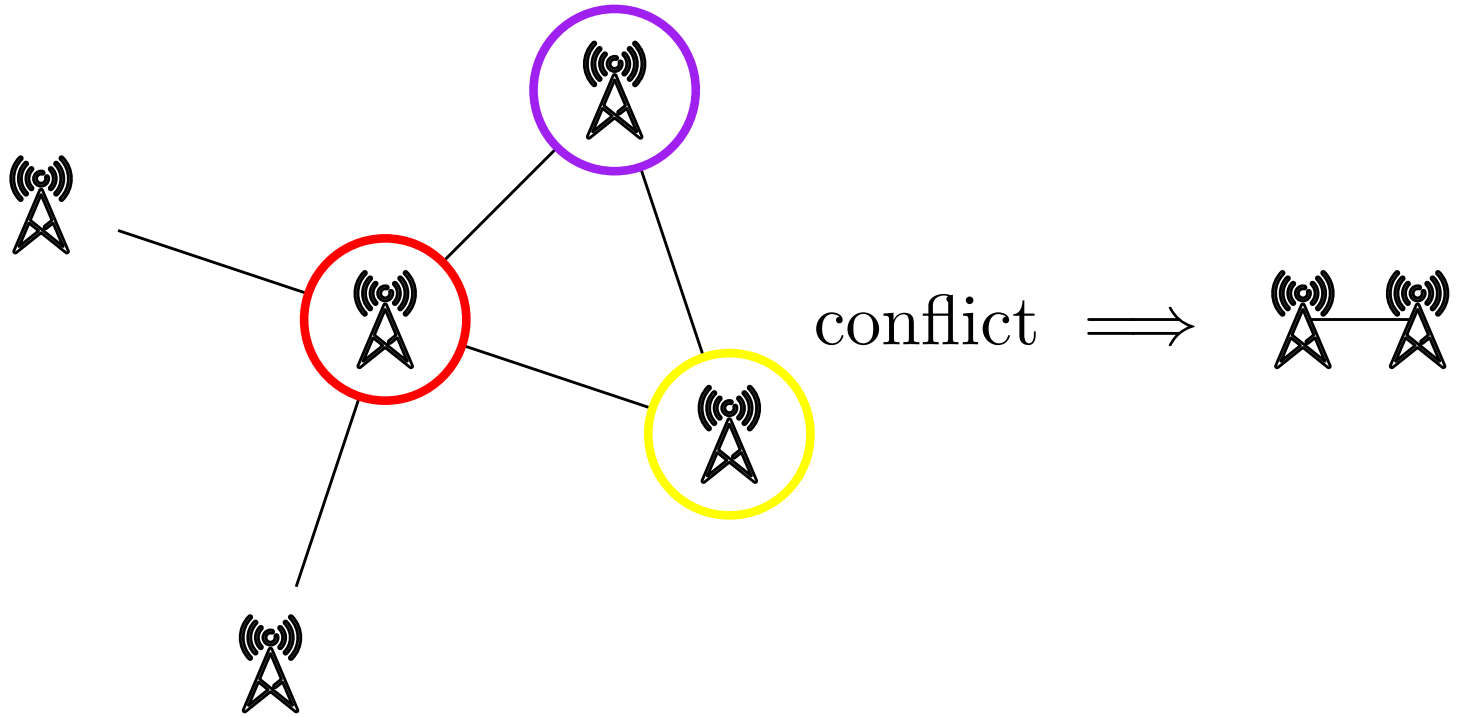
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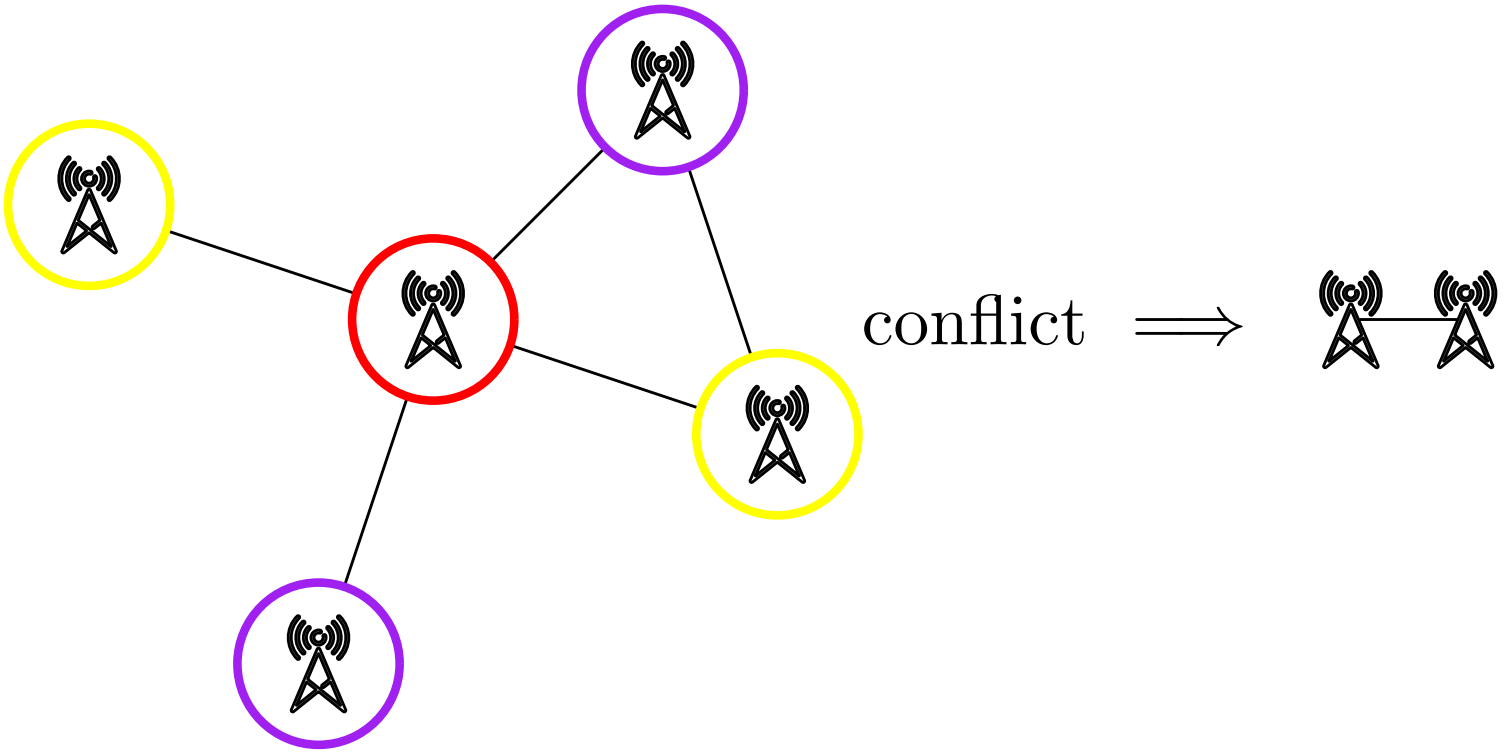
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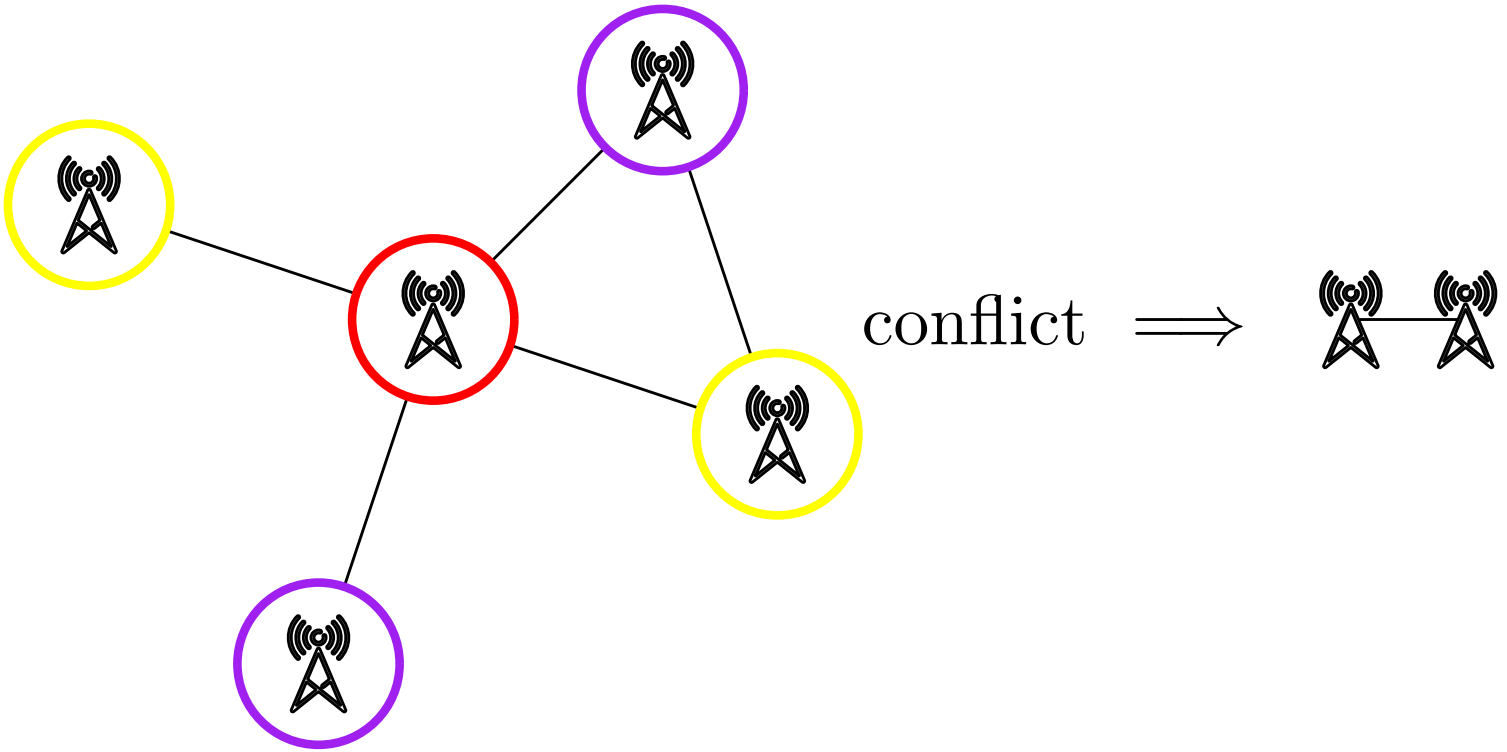

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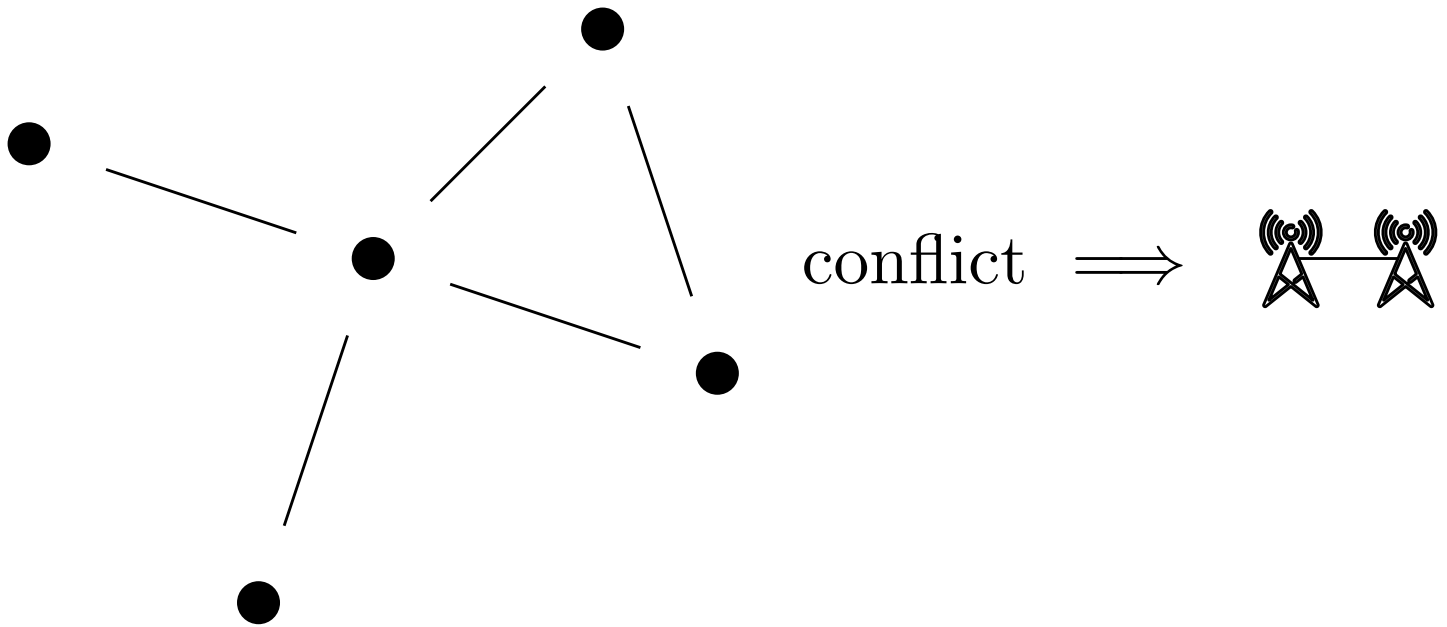

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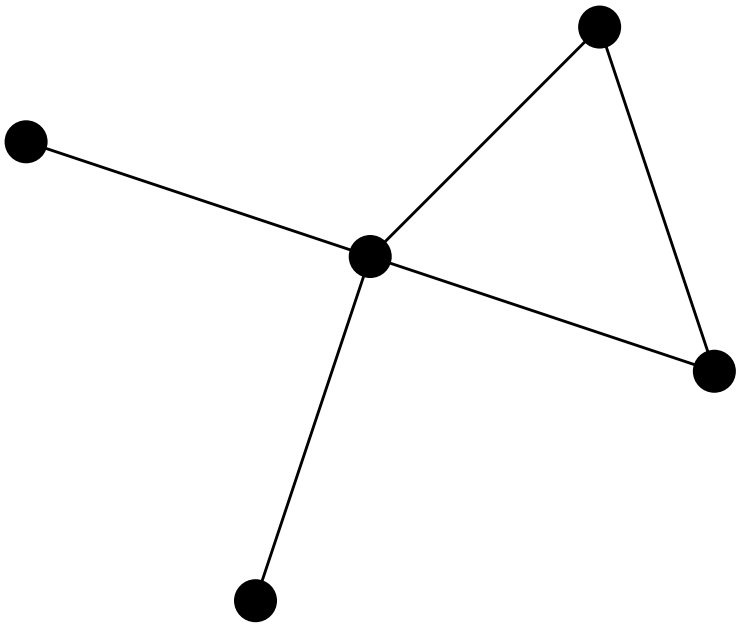
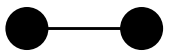


conflict $\Longrightarrow$ 

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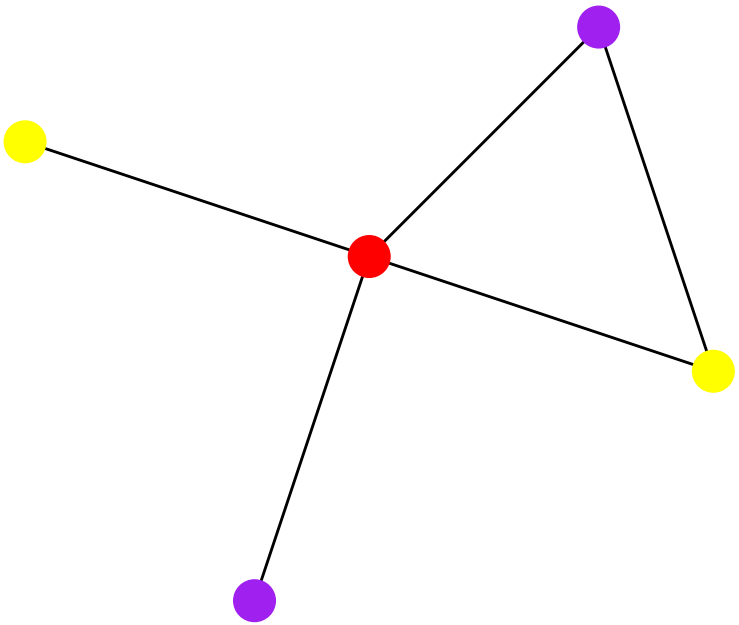
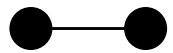


conflict $\Longrightarrow$ 

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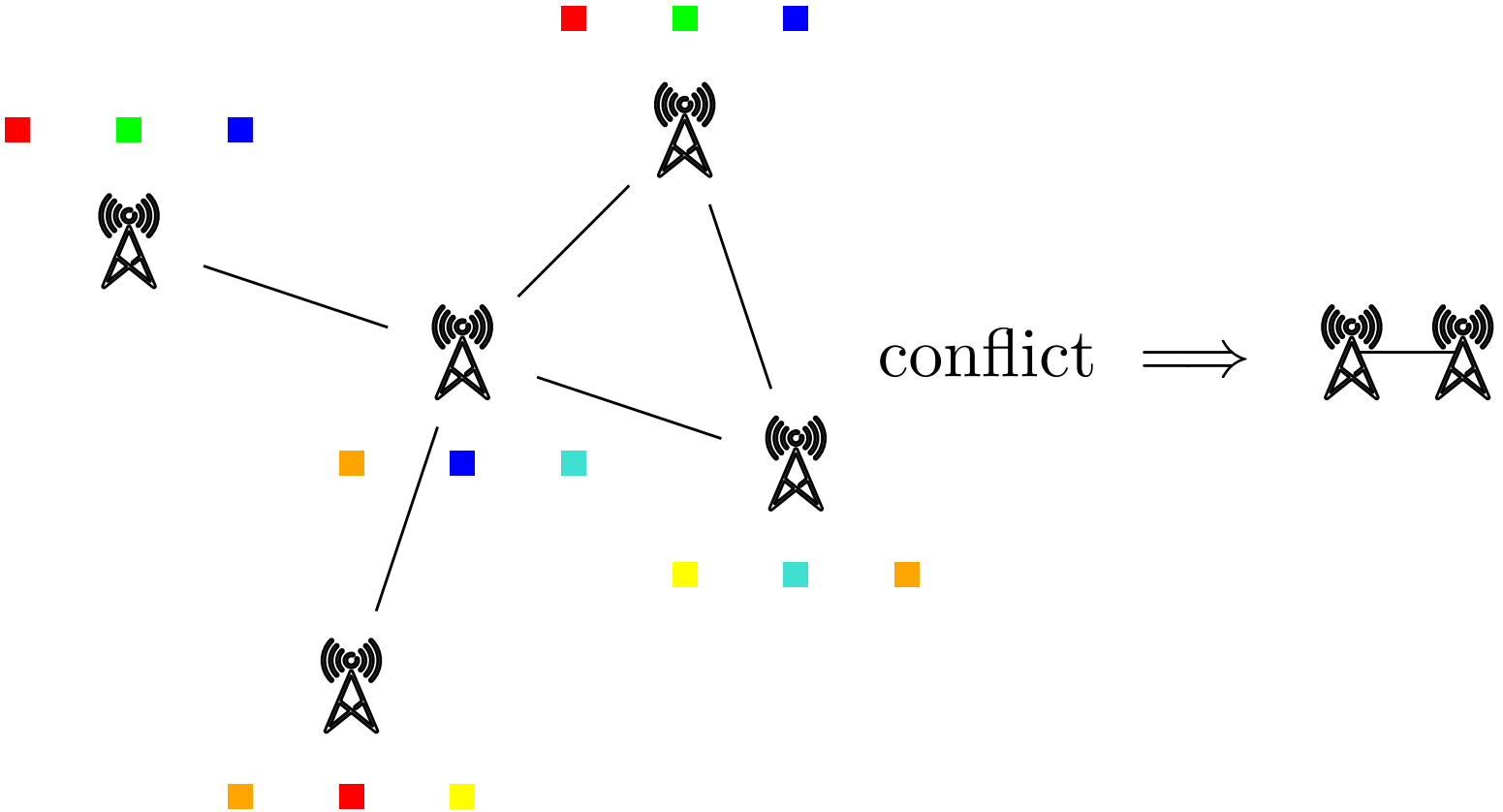

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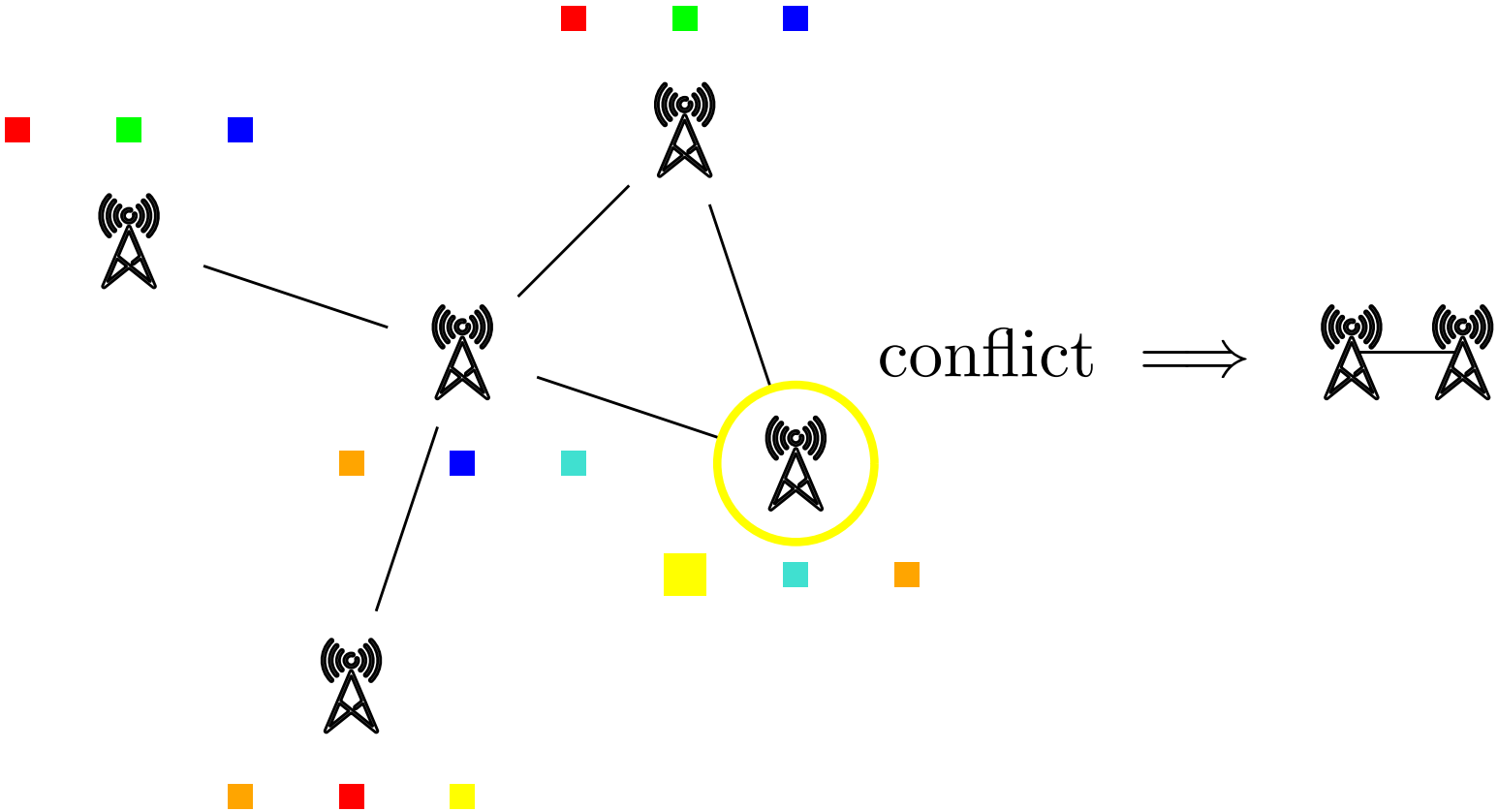

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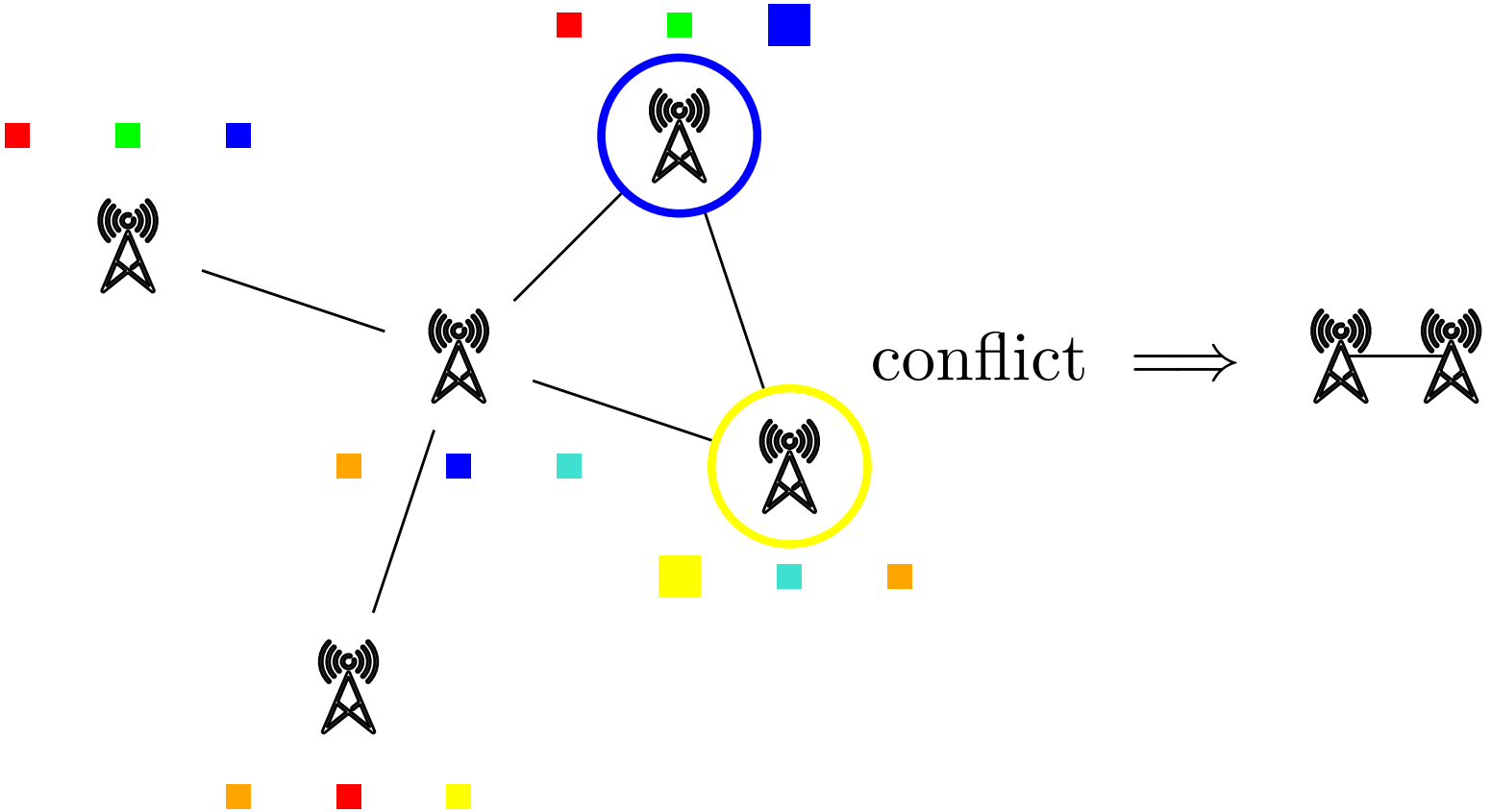

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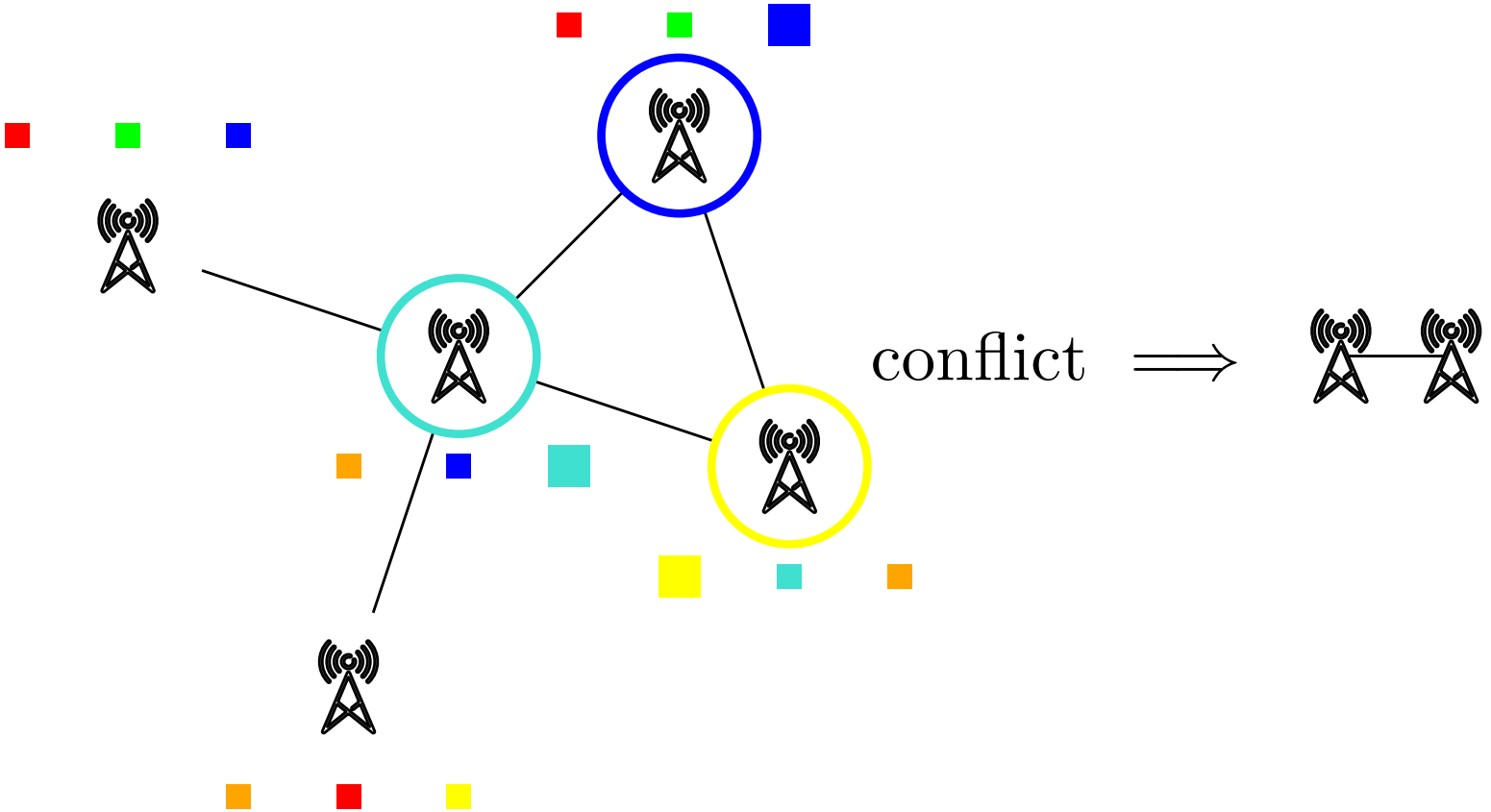

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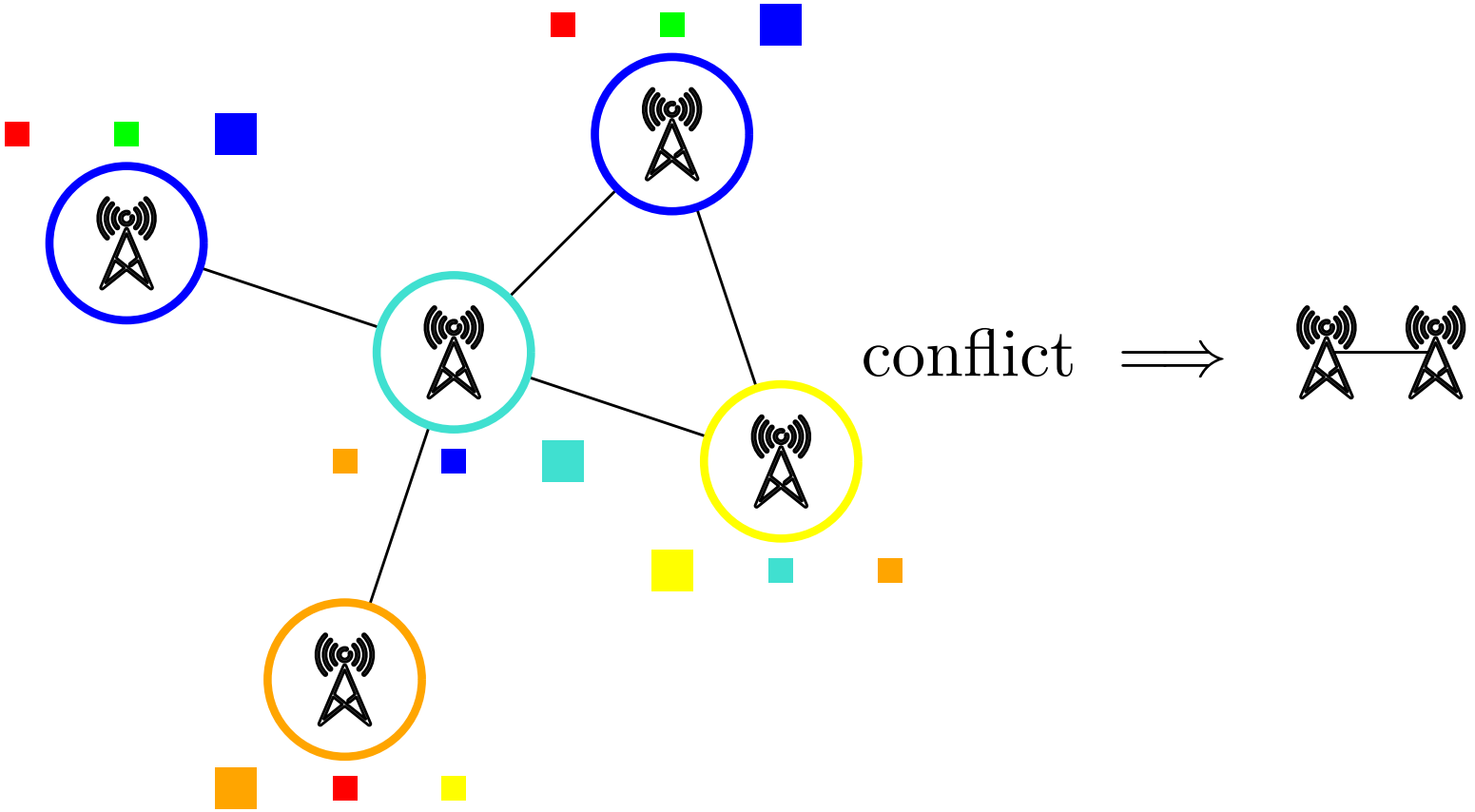

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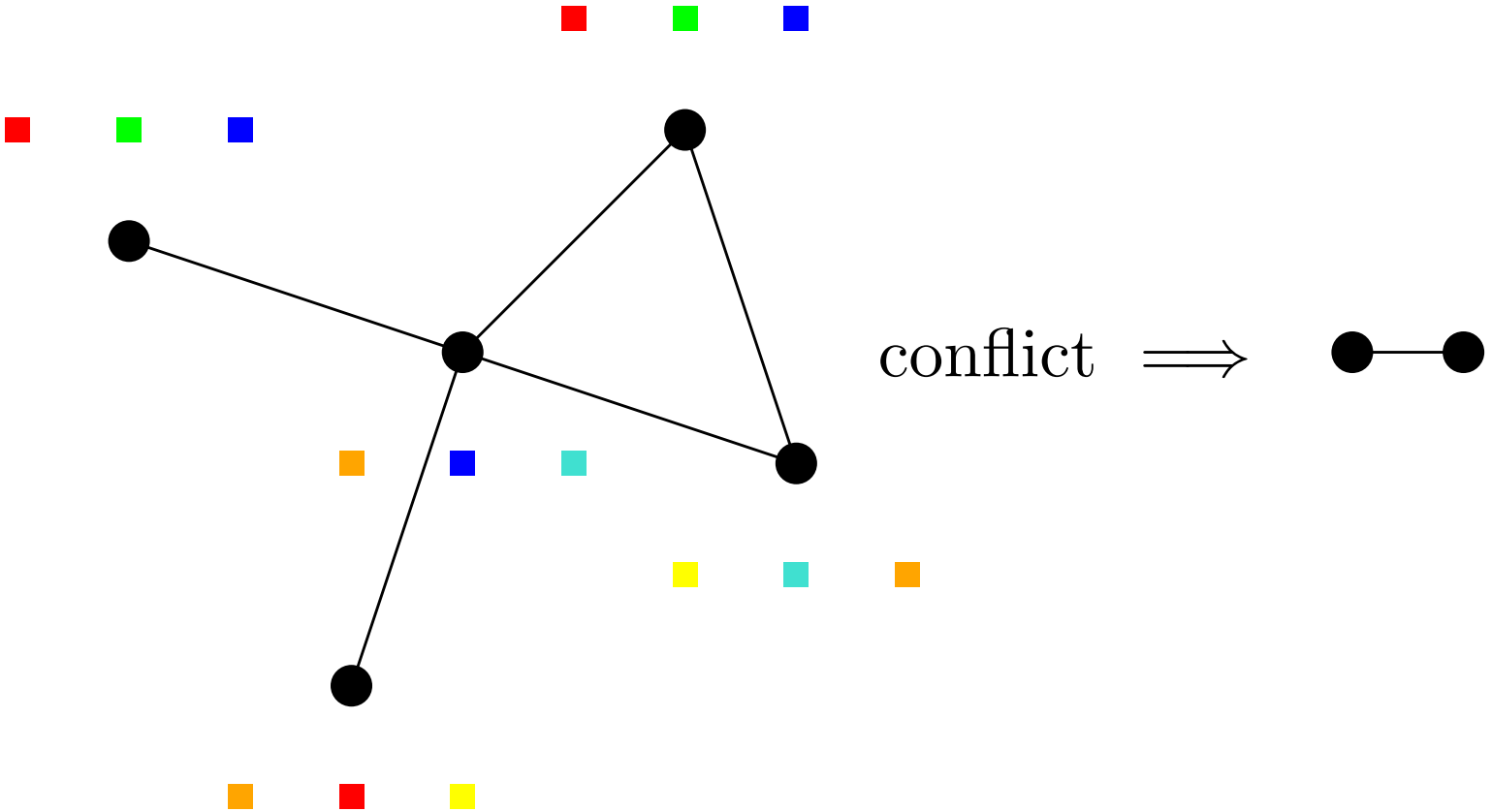

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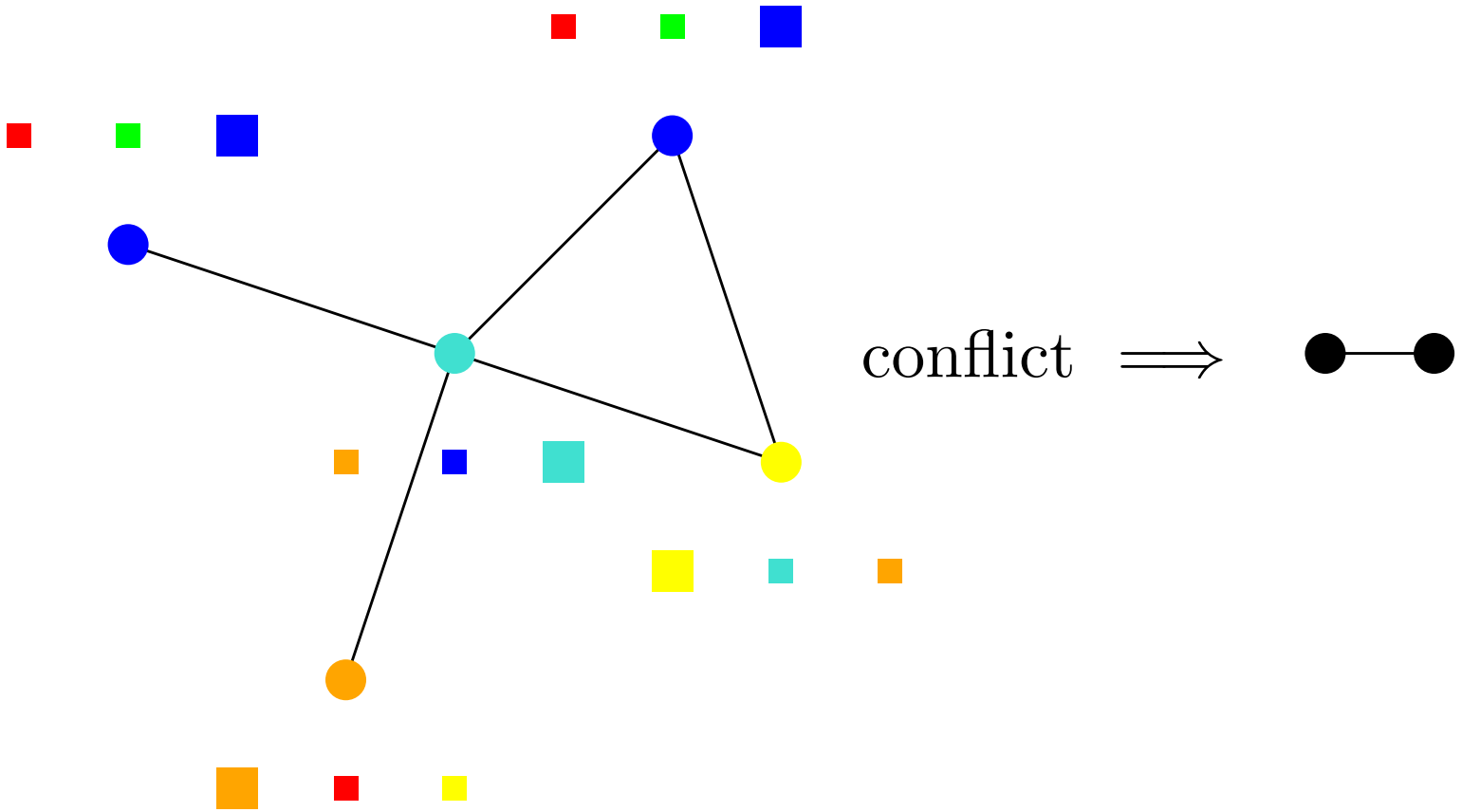

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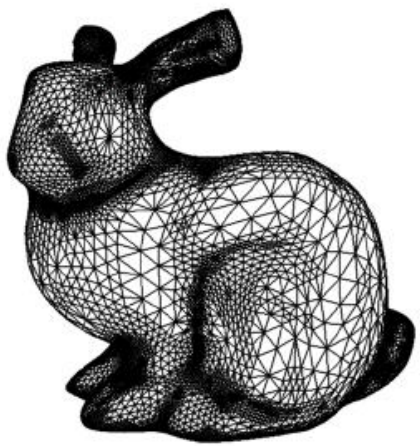
They also have many interactions with:

- computer science, physics
- combinatorial optimization
- computational geometry
- ...



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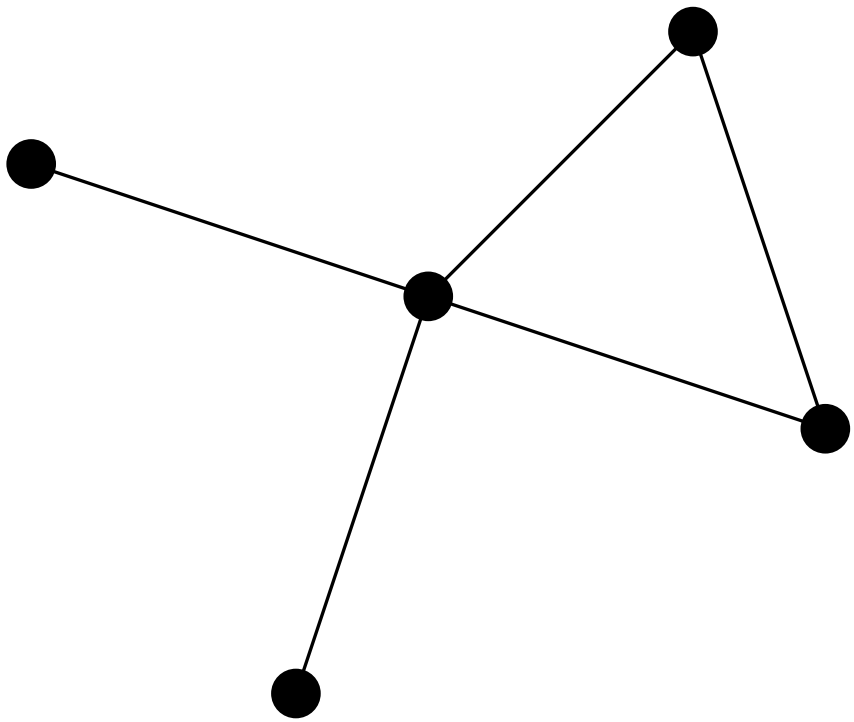

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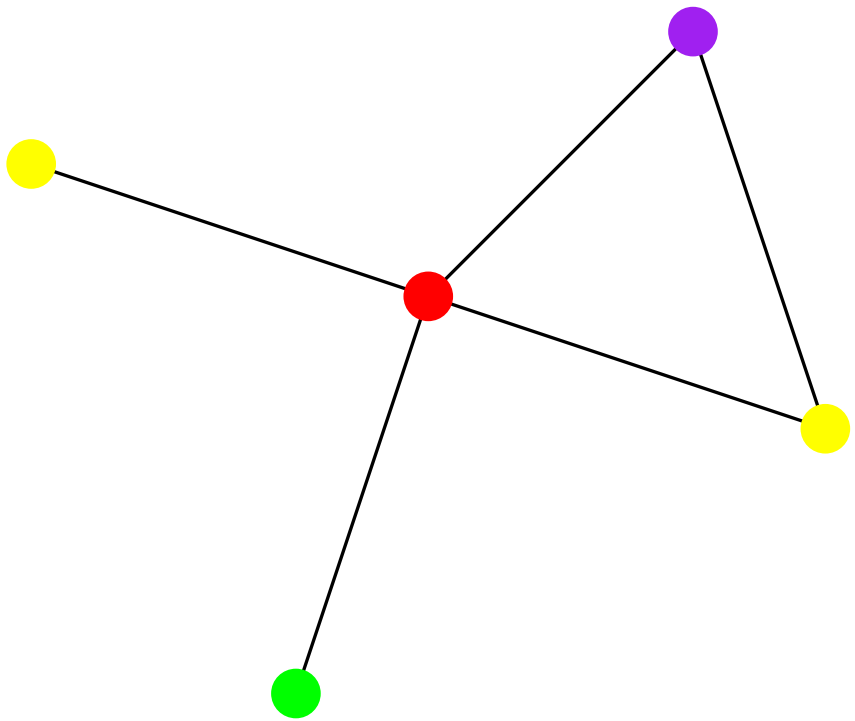

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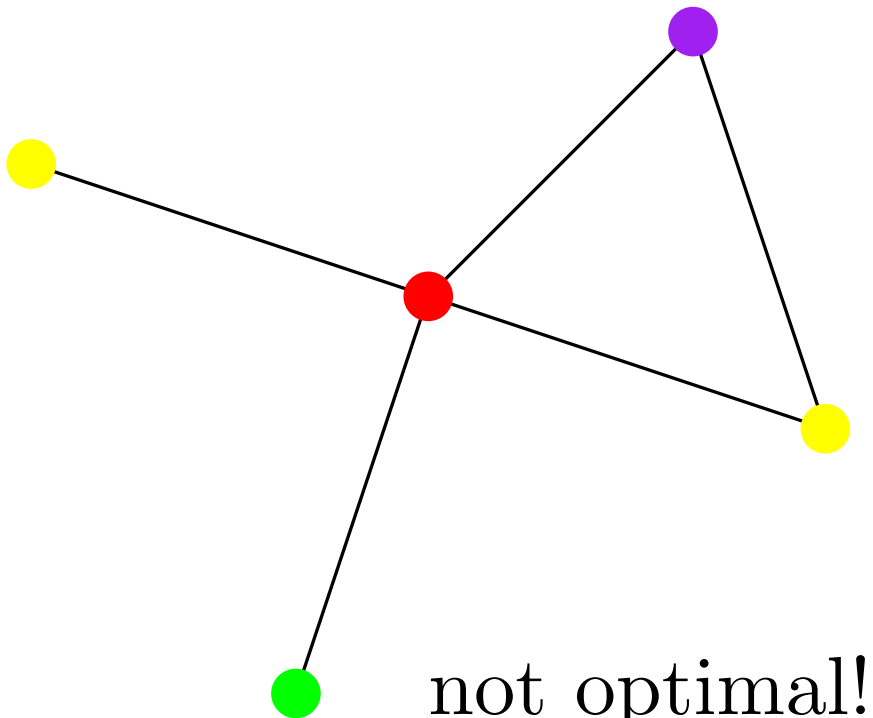

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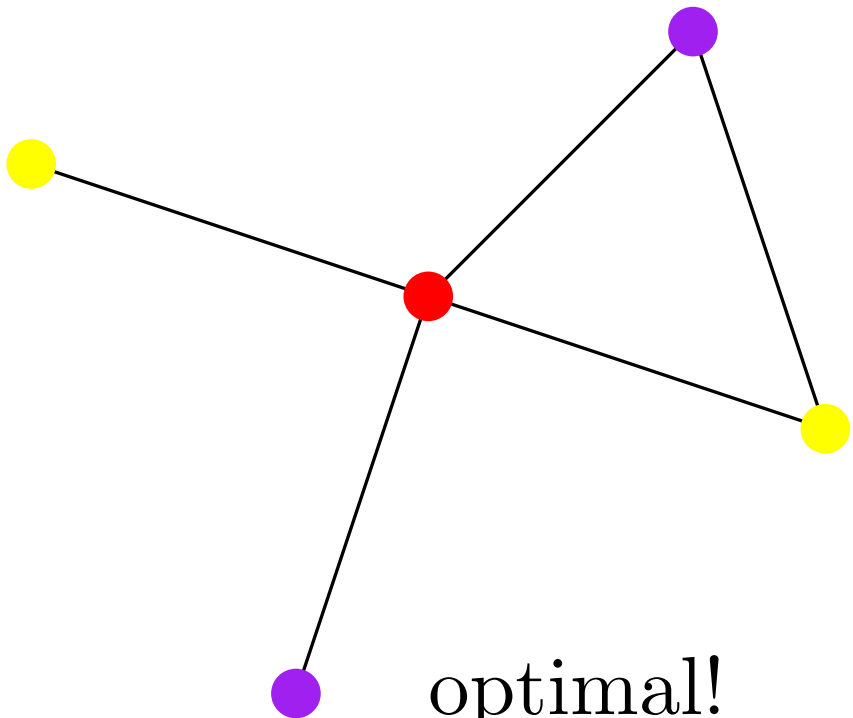

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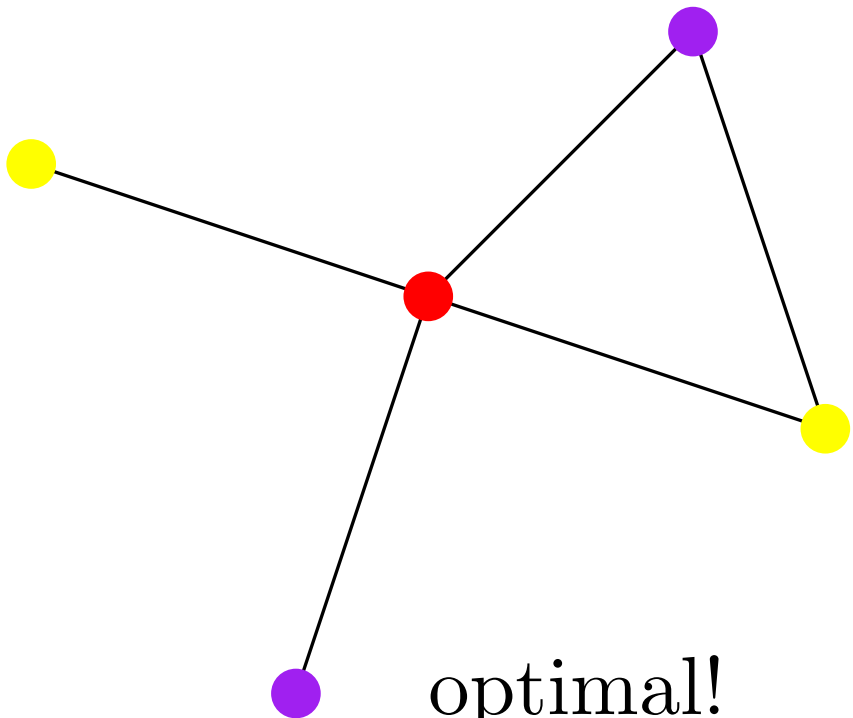

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But also in structural graph theory.

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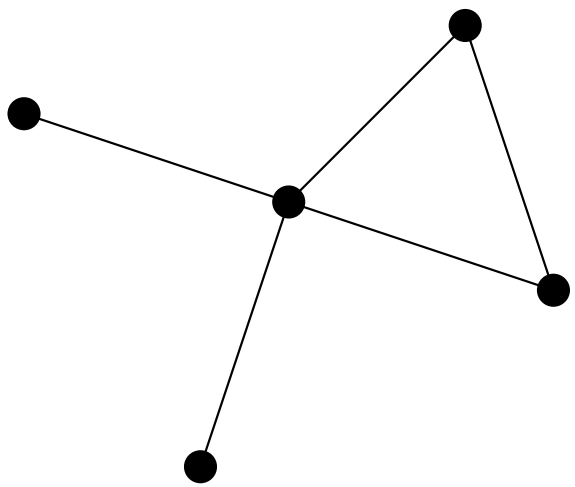

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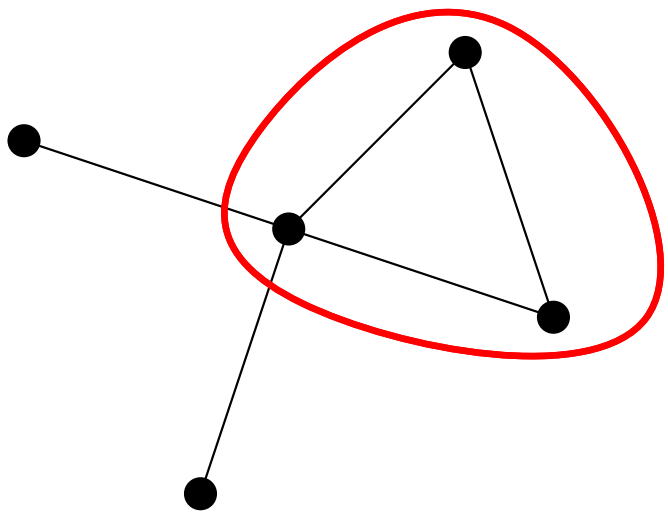

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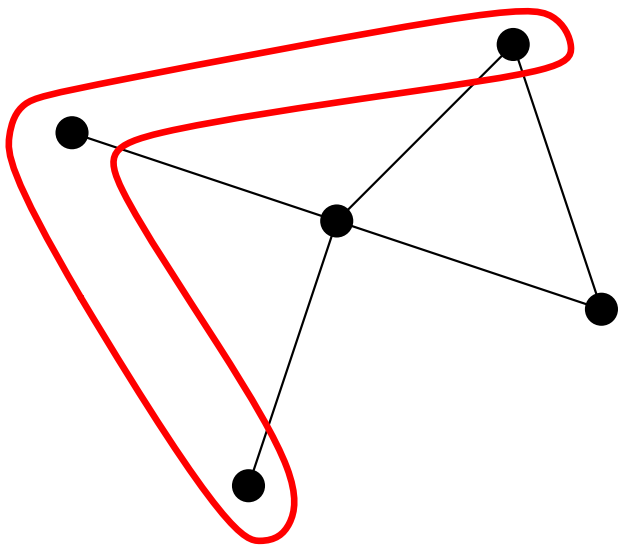
A **stable set** is a set of pairwise non-adjacent vertices. The size of the largest stable set is denoted by $\alpha(G)$.



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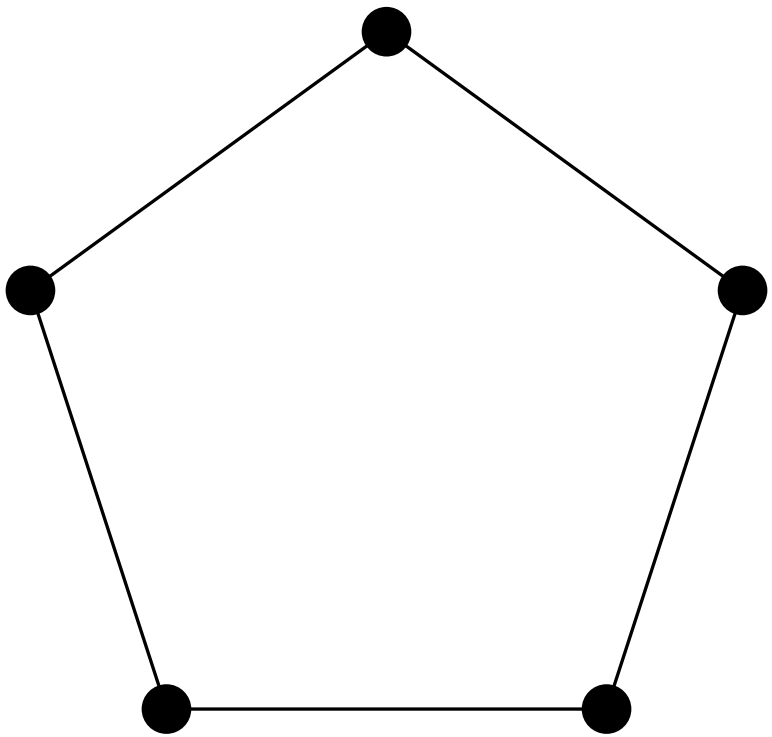

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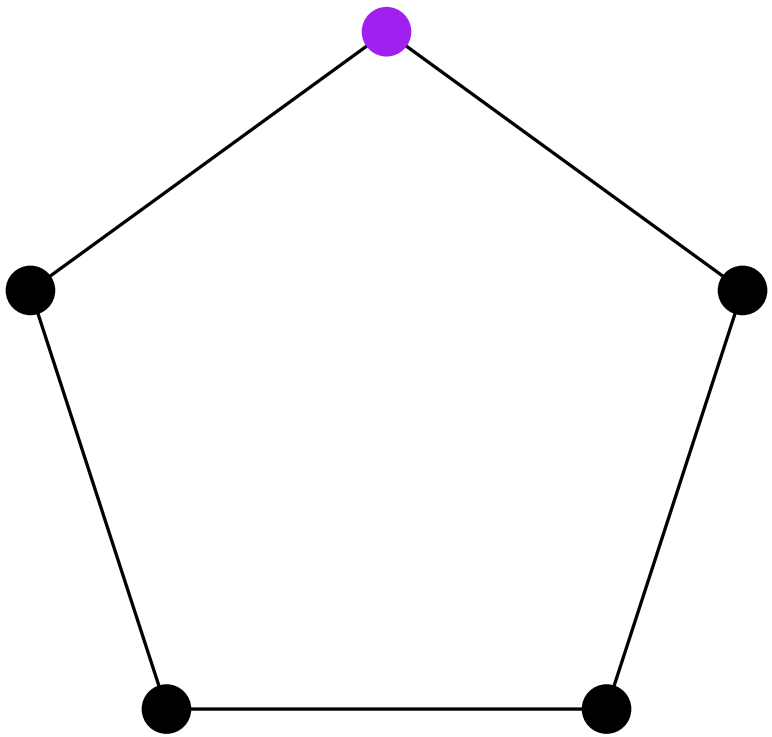

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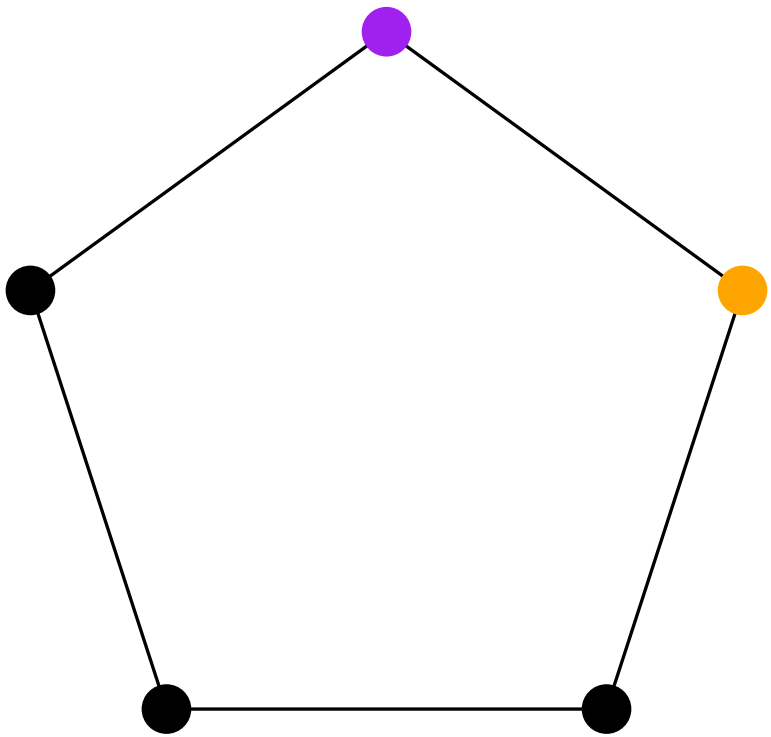

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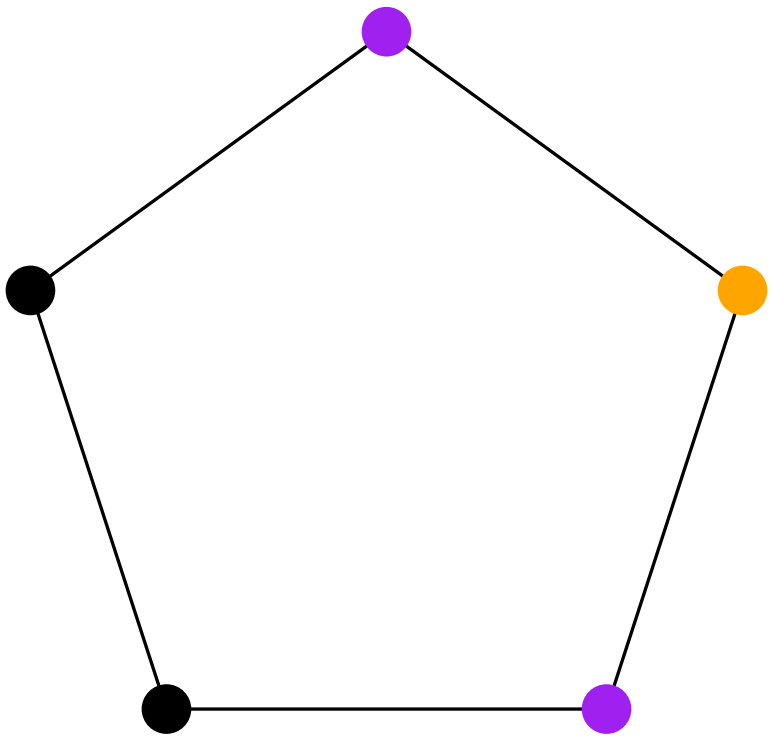

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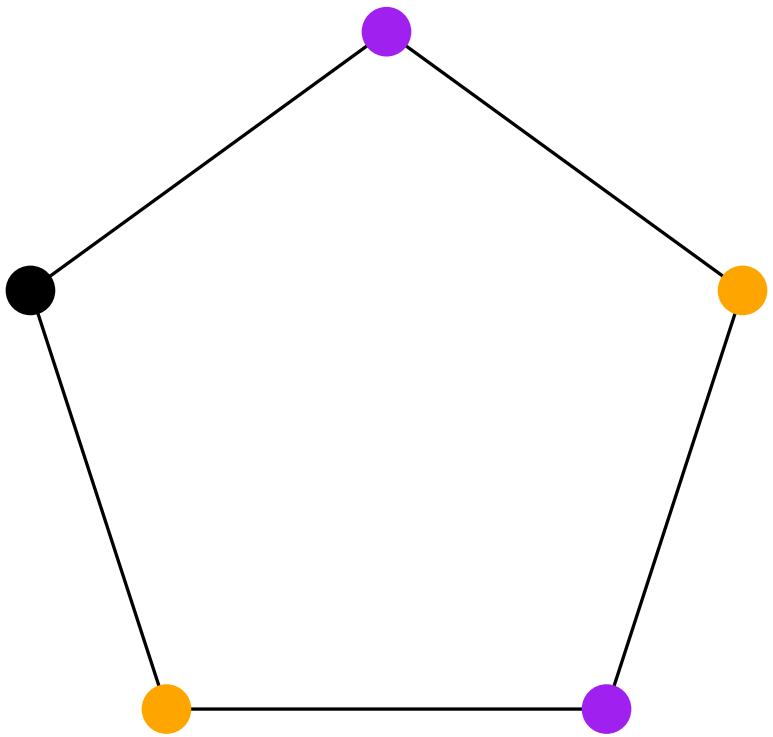

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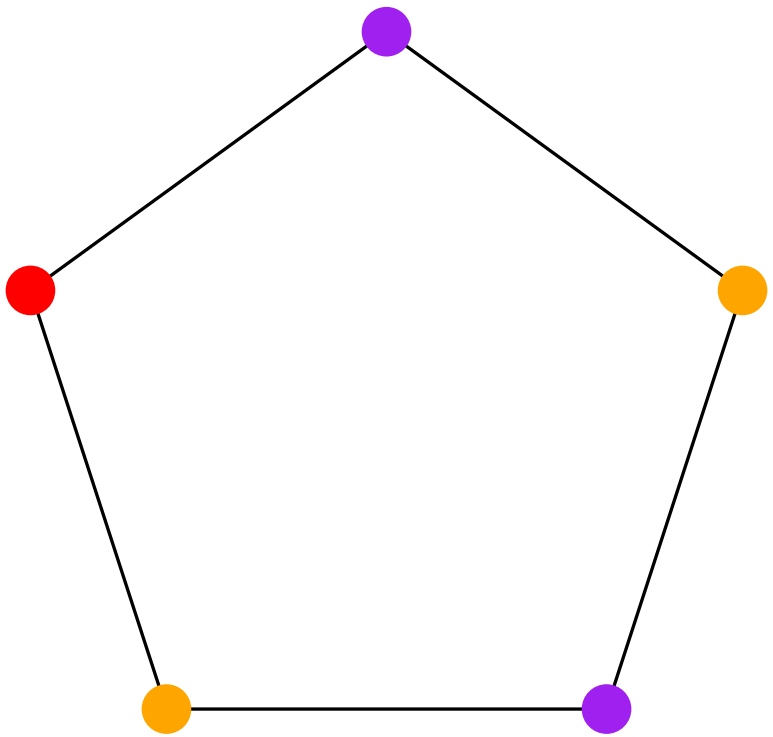

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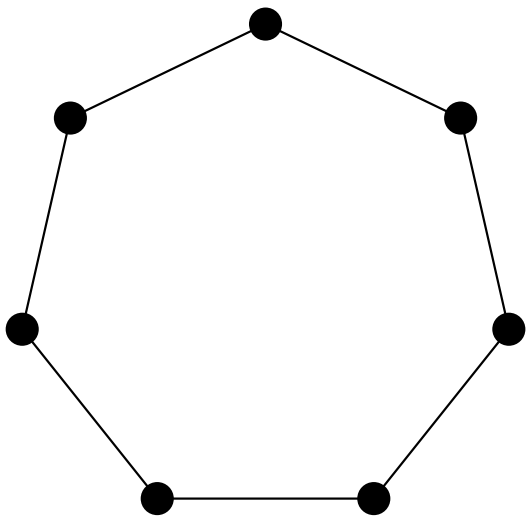

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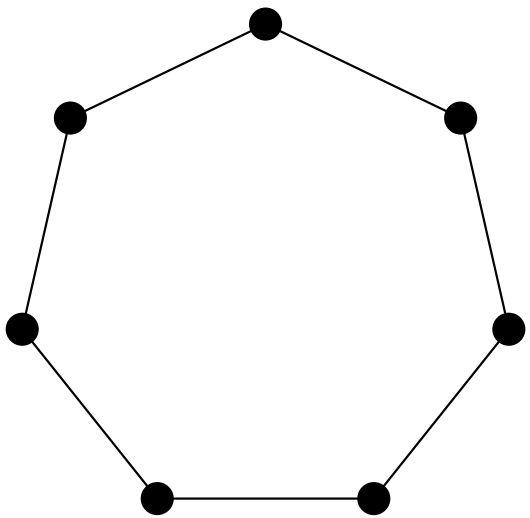

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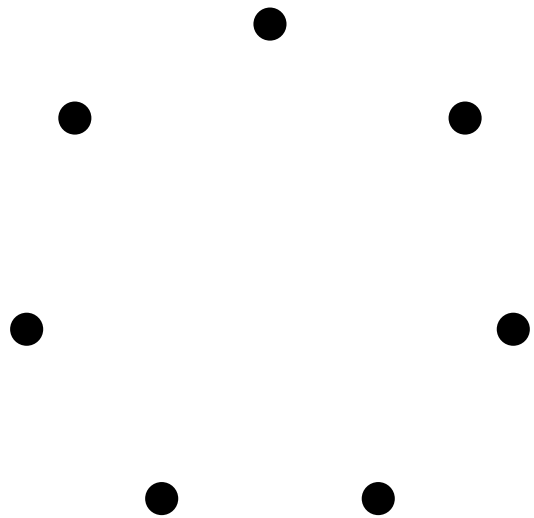

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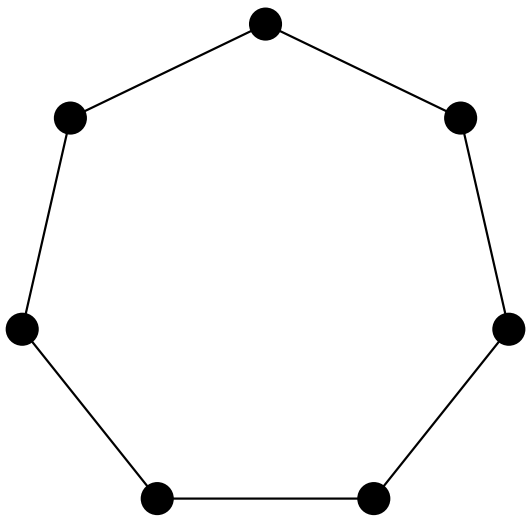

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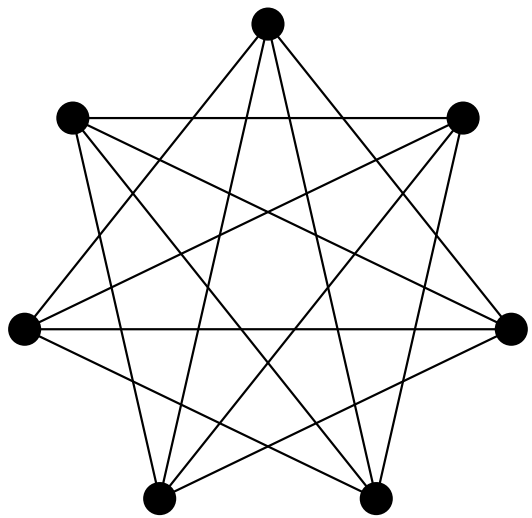

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Conjectured by Berge in 1961.

Theorem [*Chudnovsky, Robertson, Seymour, Thomas 2002*]

A graph is perfect if and only if it does not contain any odd hole or odd anti-hole as an induced subgraph.

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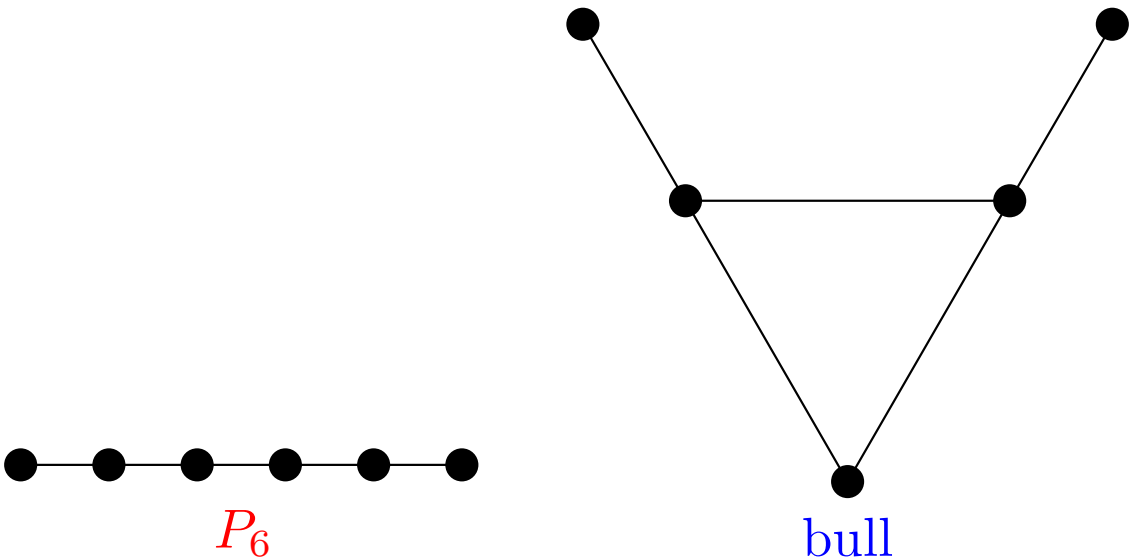
What about coloring a graph with a **fixed** number of colors?

Coloring (P_6, bull)-free graphs



k-coloring

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Let us take a look at **other graph classes**.

Forbidding short cycles [*Kamiński, Lozin 2007*]

For any fixed $k, g \geq 3$, the k-coloring problem is NP-Complete in the class of graphs with **girth** at least g.

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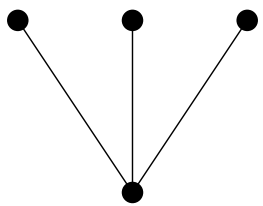

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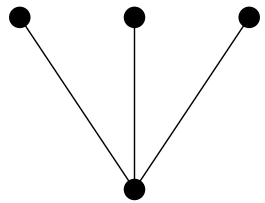

claw

Forbidding cycles or claws

For any fixed $k \geq 3$ and H a forbidden induced subgraph that is **not a collection of paths**, deciding whether a H-free graph is k-colorable is NP-Complete.

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k-coloring of P_ℓ-free graphs.

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$\ell \backslash k$	≤ 2	3	4	≥ 5
≤ 4	P	P	P	P
5	P	P	P	P
6	P	P	?	NPC
7	P	P	NPC	NPC
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$\cdots$

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(6)	*P*	*P*	(?)	*NPC*
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⋮

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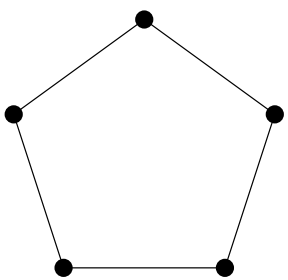
- (P_6, C_5)-free graphs [*Chudnovsky et al. 2014*].



C_5

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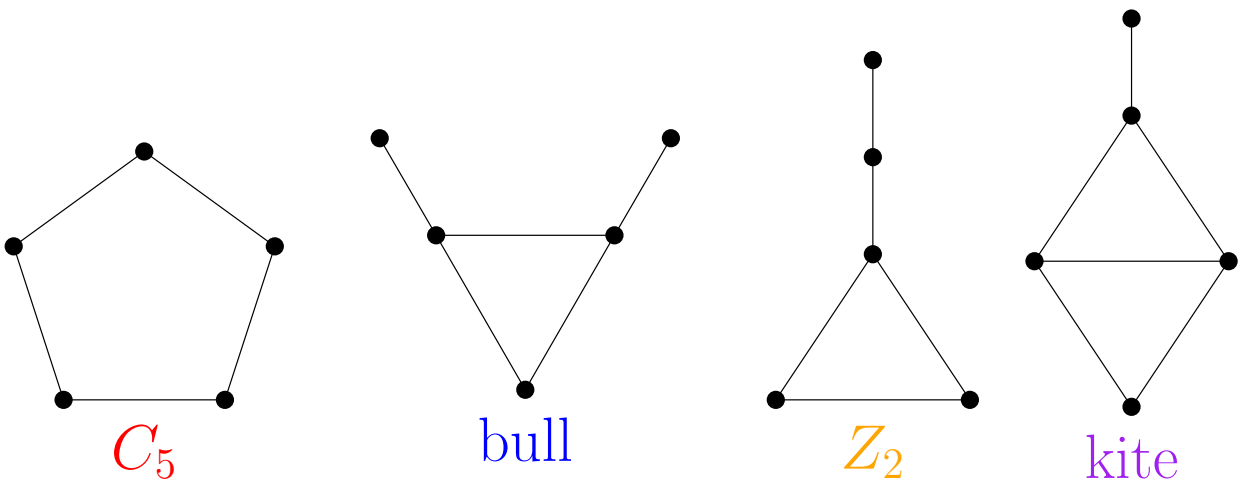

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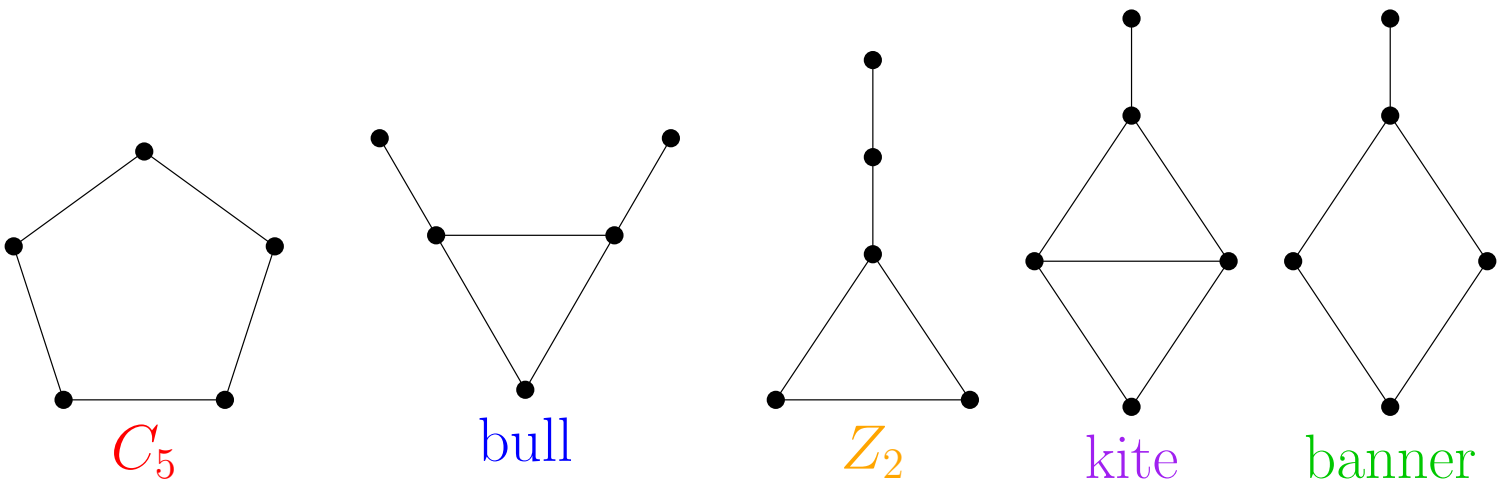

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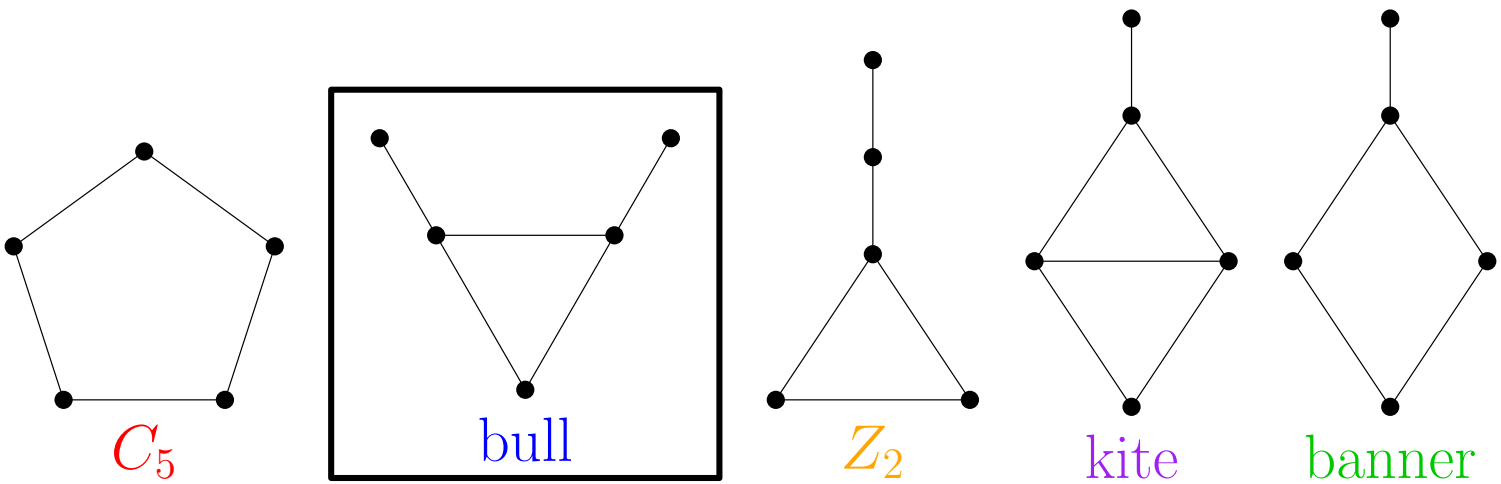

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A **homogeneous set** is a set $S \subseteq V(G)$ such that every vertex in $V(G) \setminus S$ is either complete to S or anti-complete to S.

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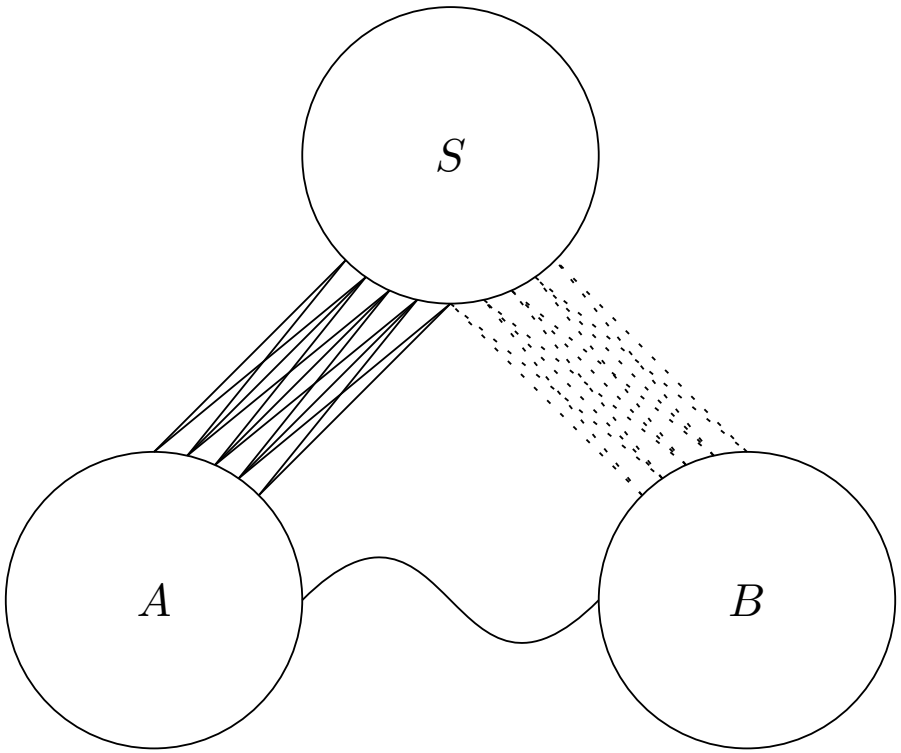

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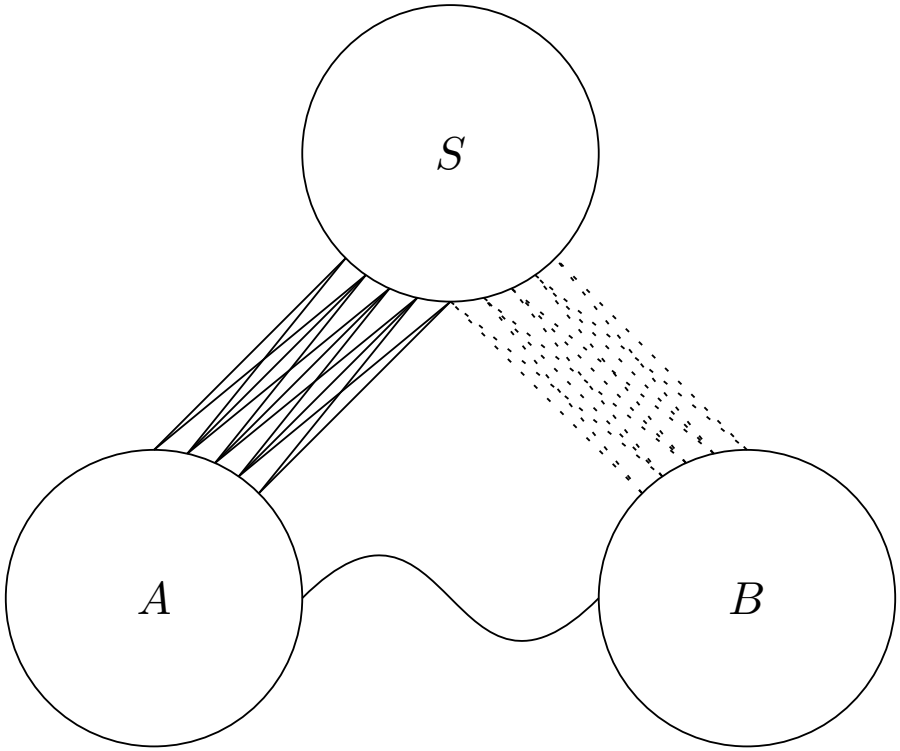

Quasi-prime graph

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Lemma

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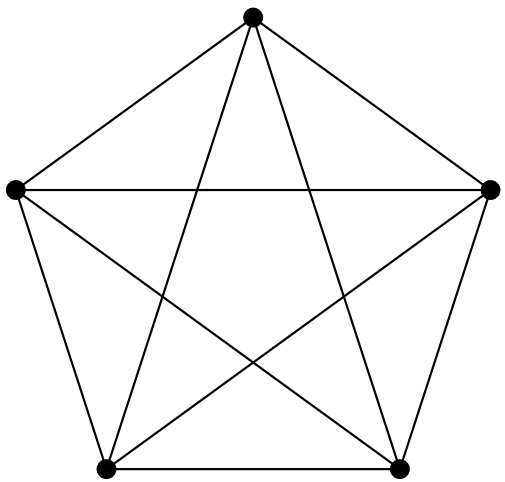
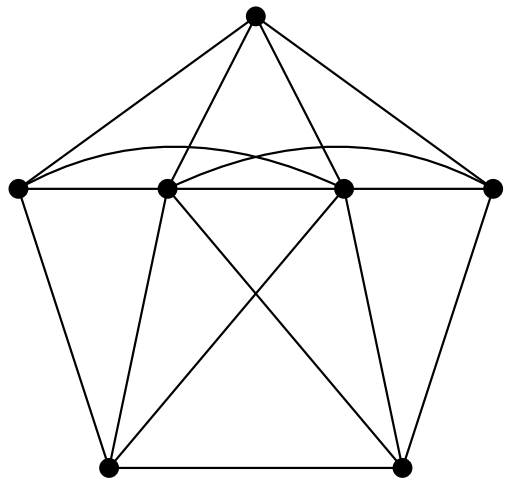


K_5



double-wheel

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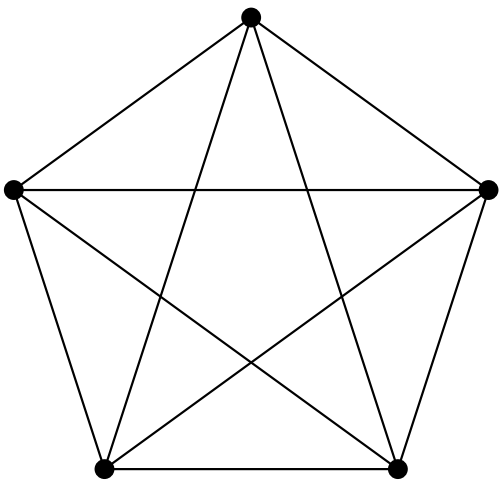

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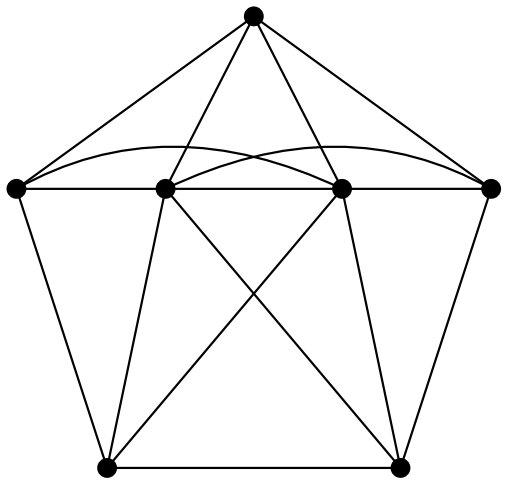

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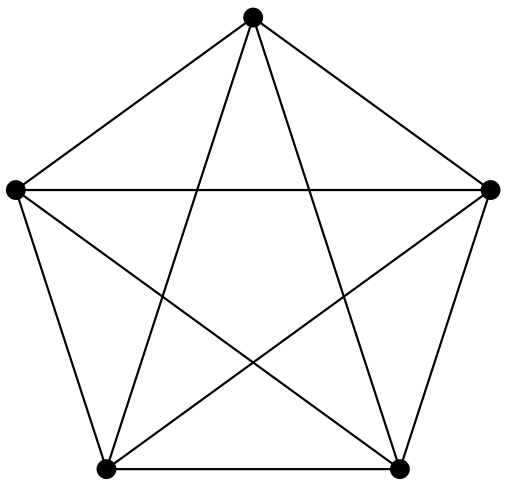

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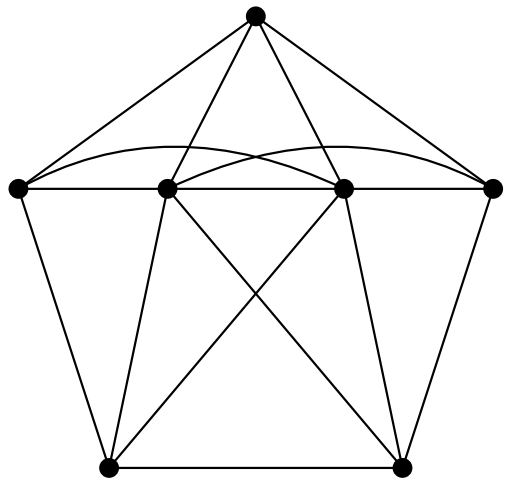

double-wheel

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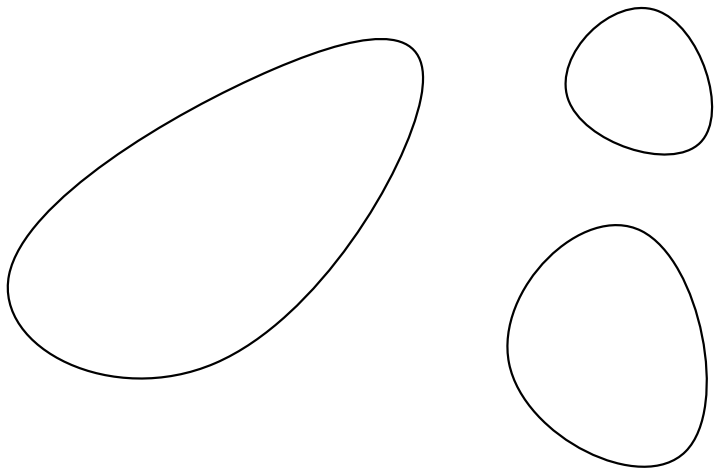

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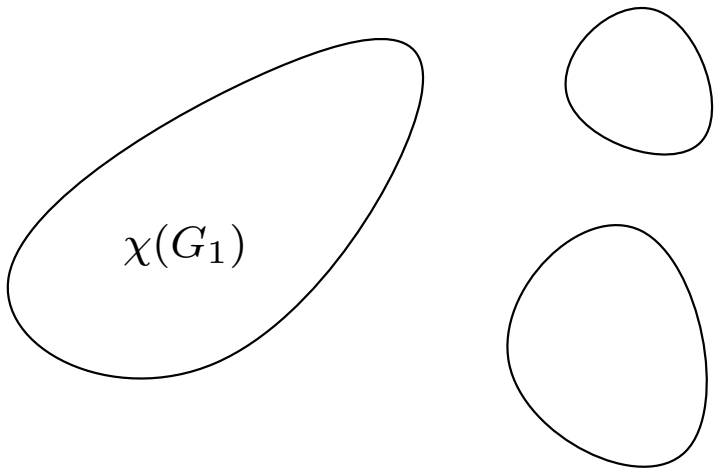

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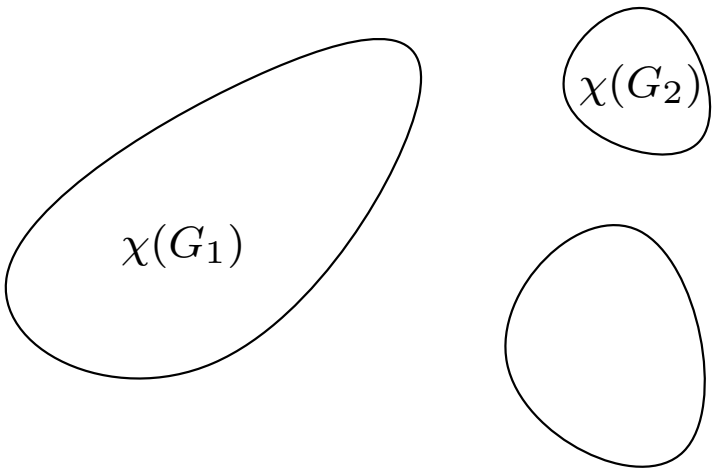

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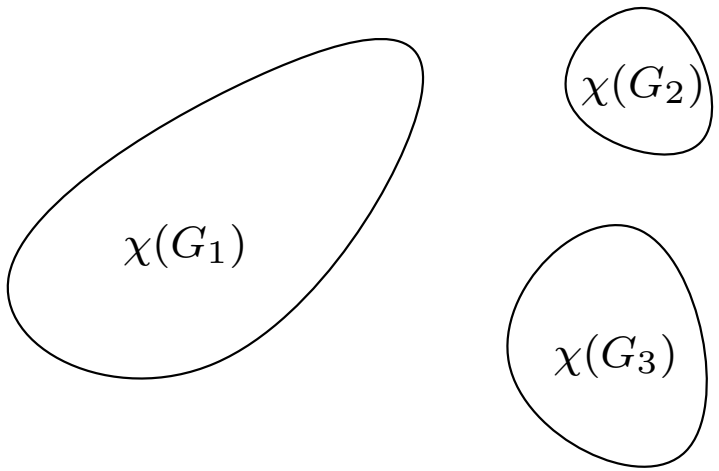

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Keep the maximum over all $\chi(G_i)$.

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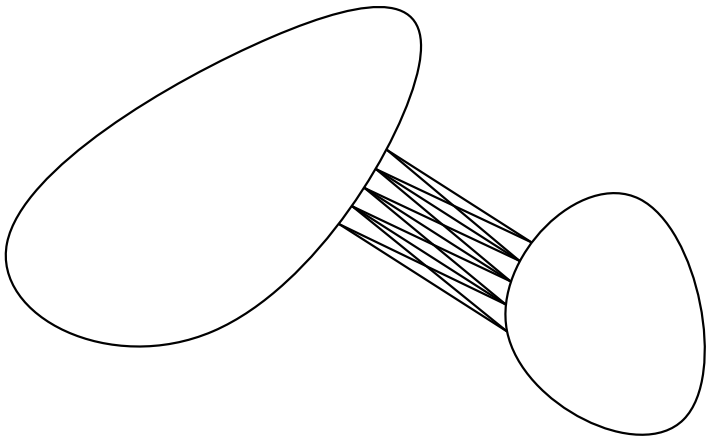

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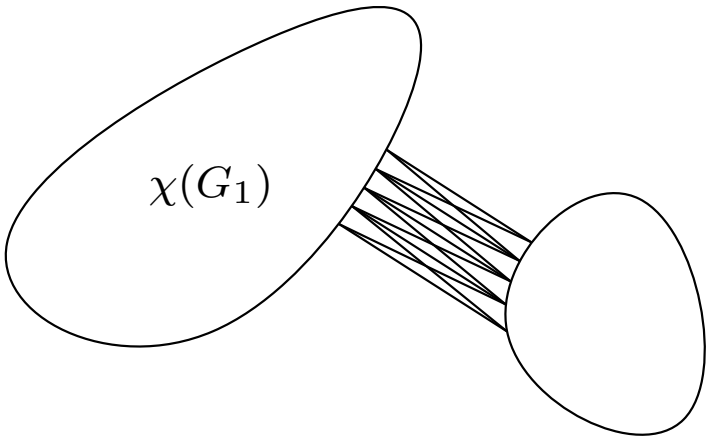

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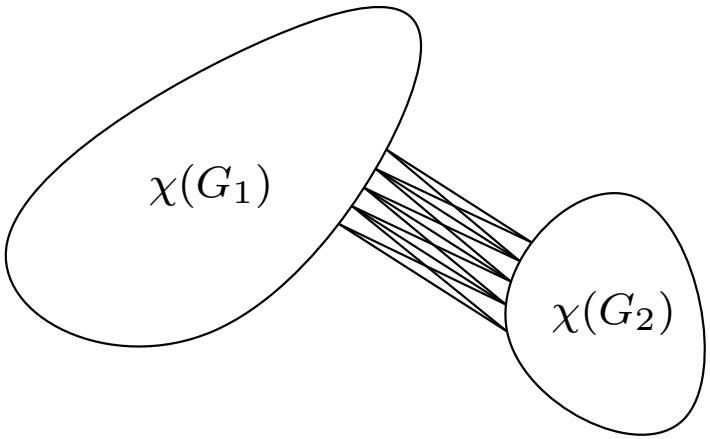

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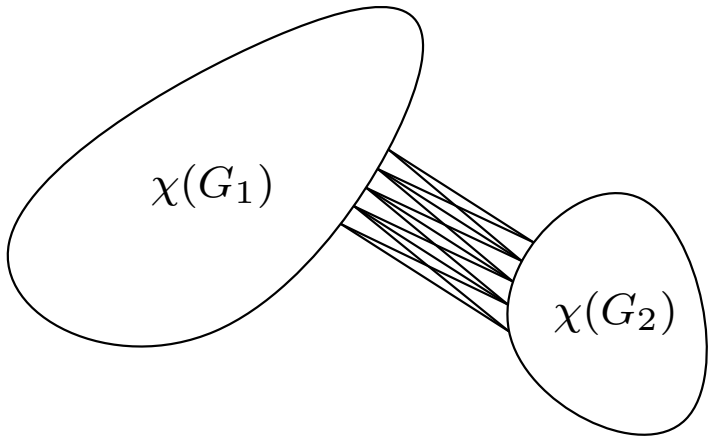

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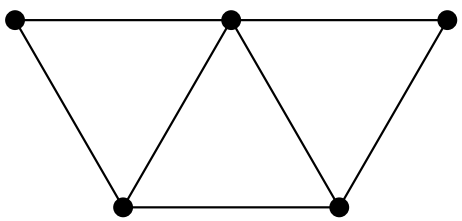

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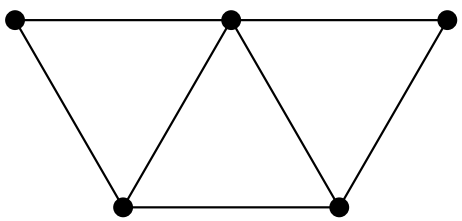

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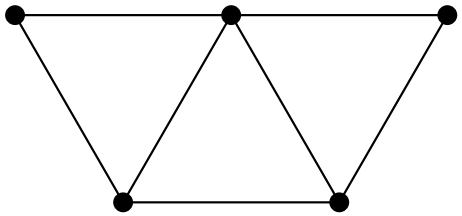

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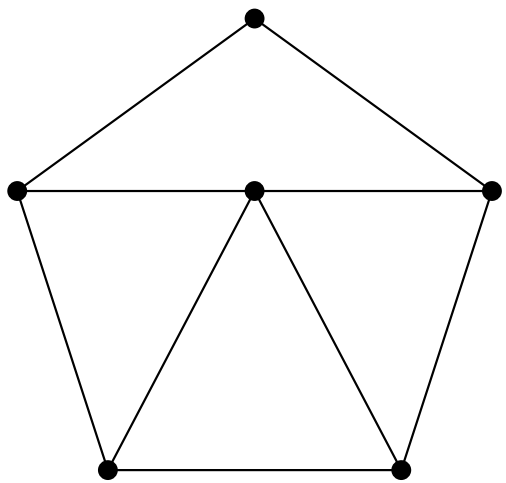

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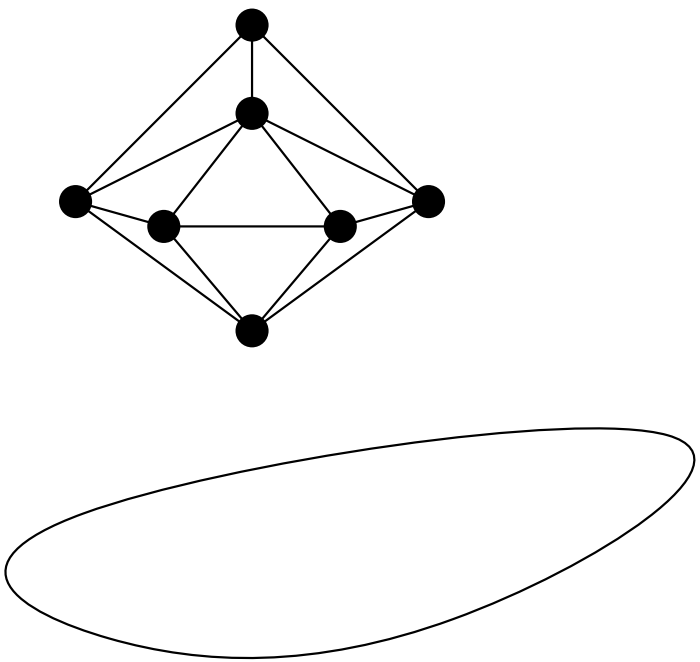

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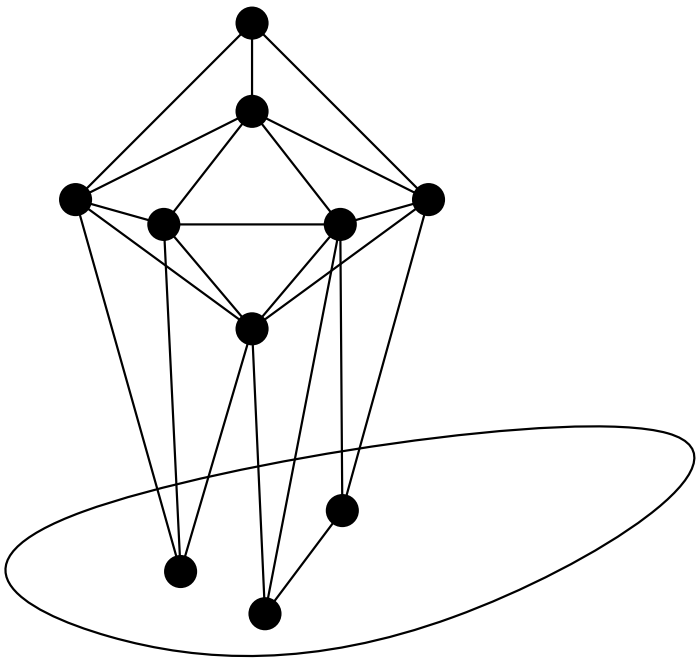

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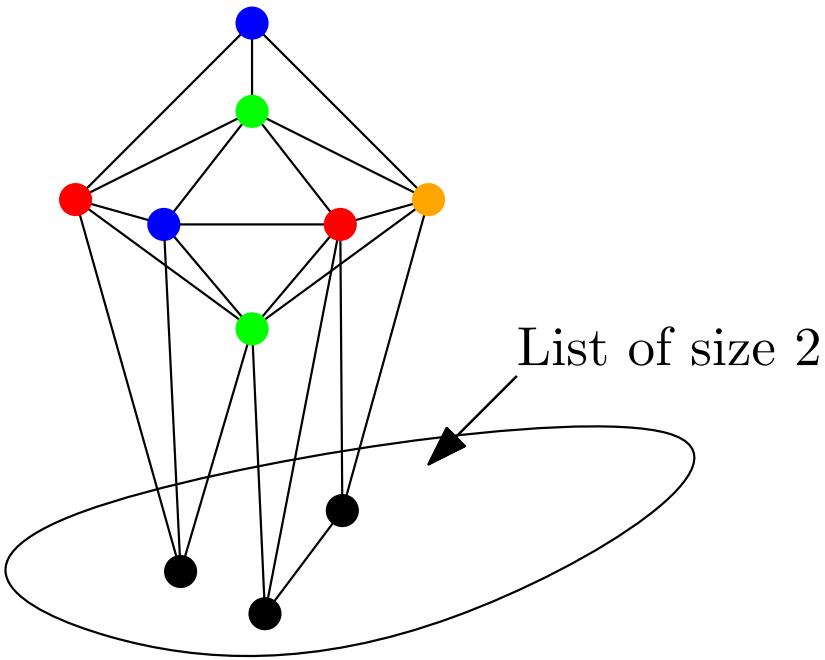

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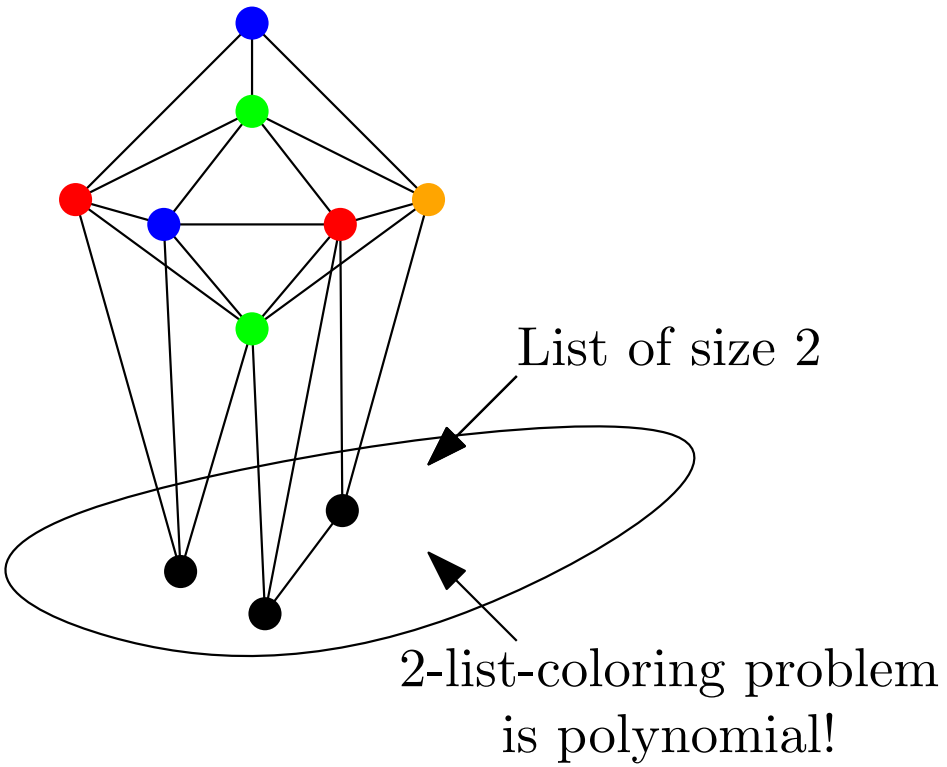

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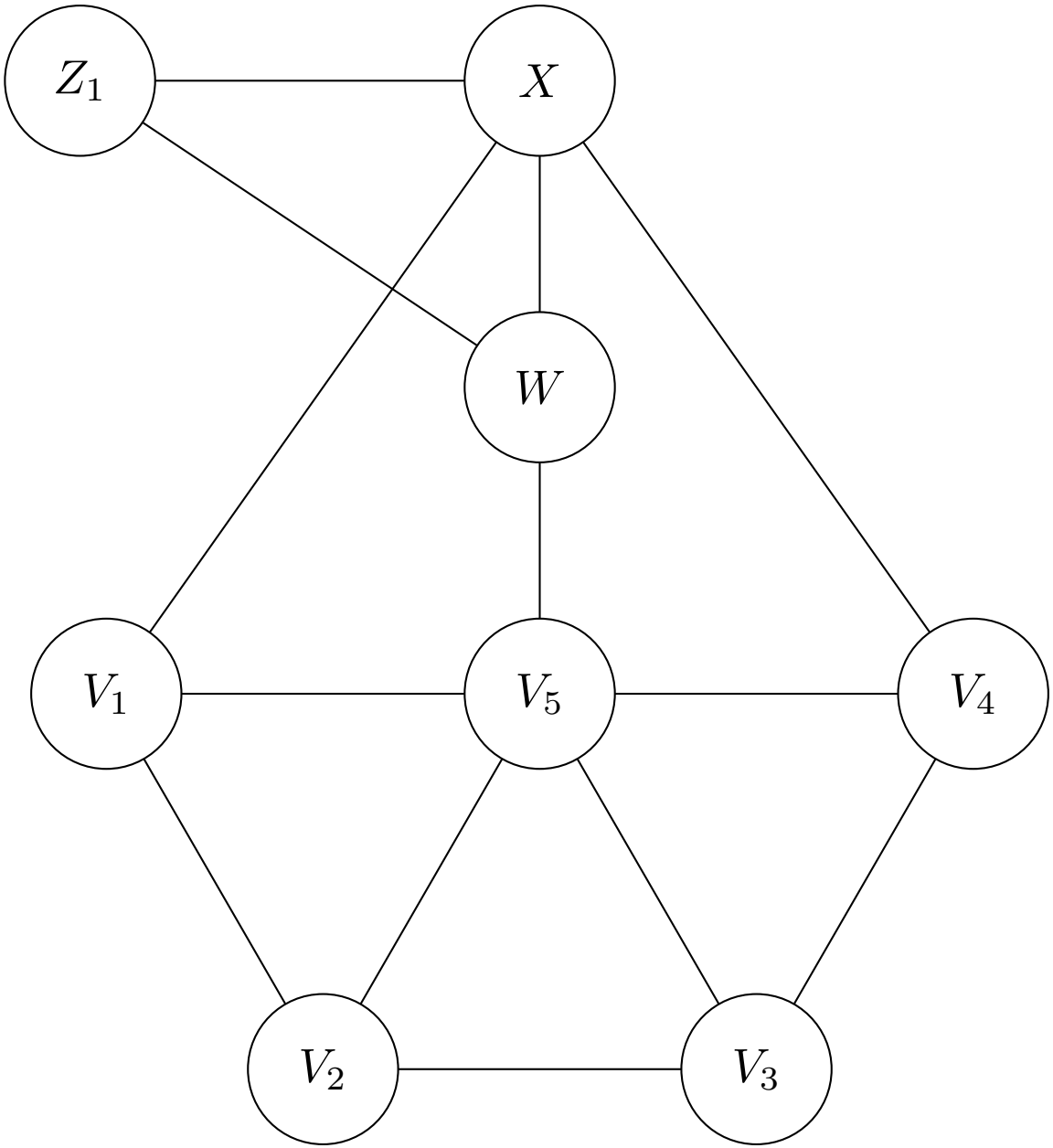
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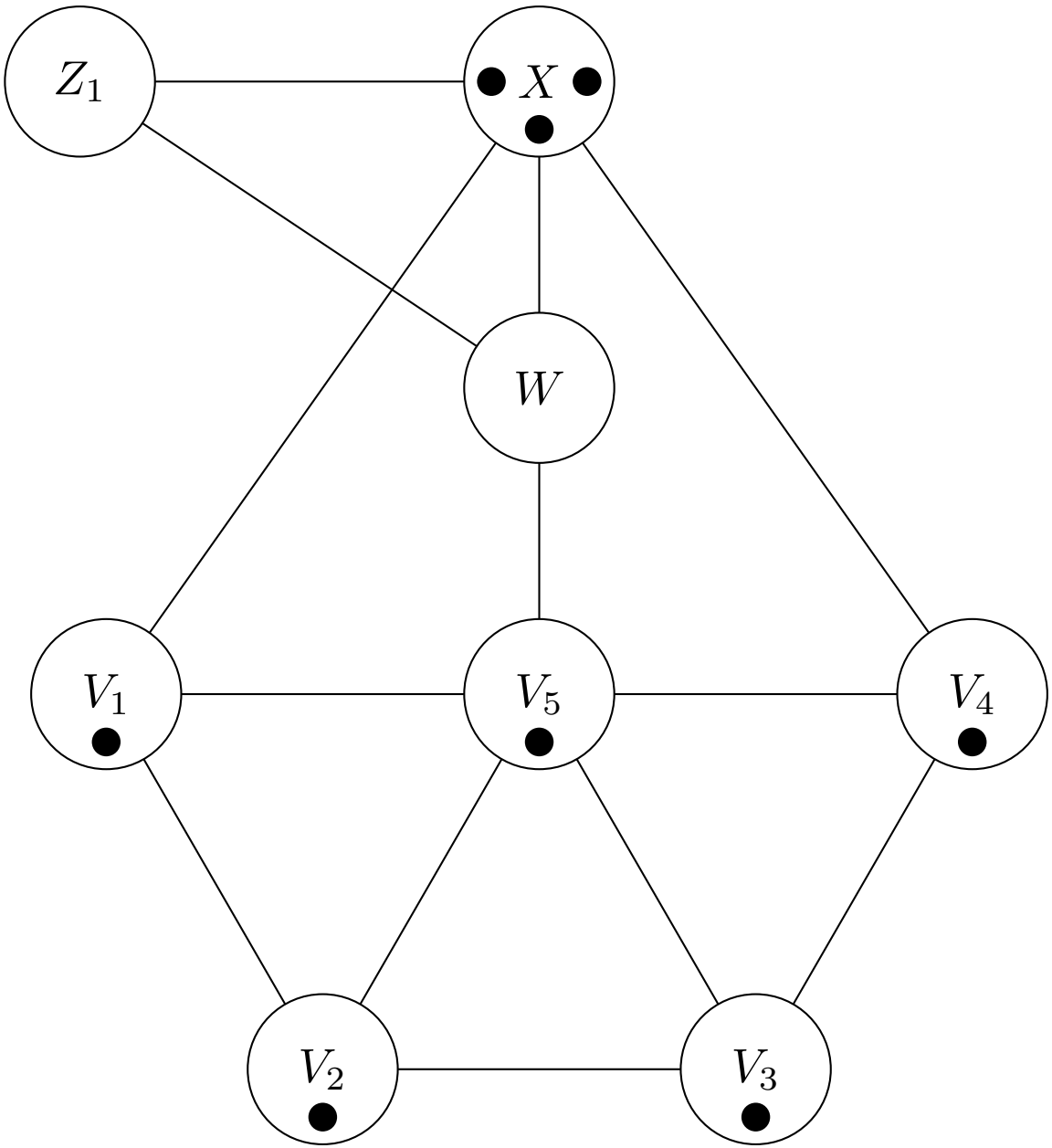
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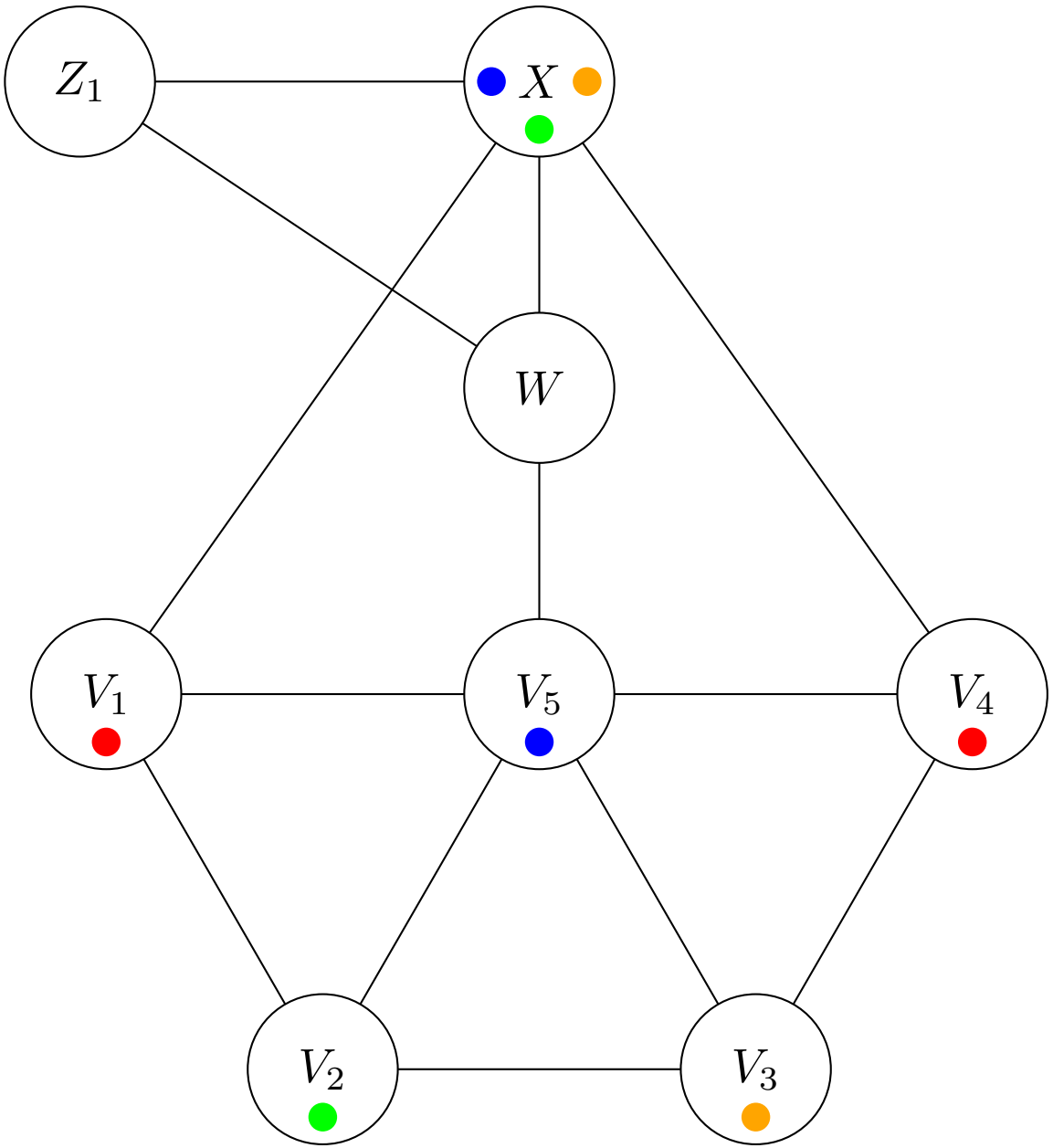
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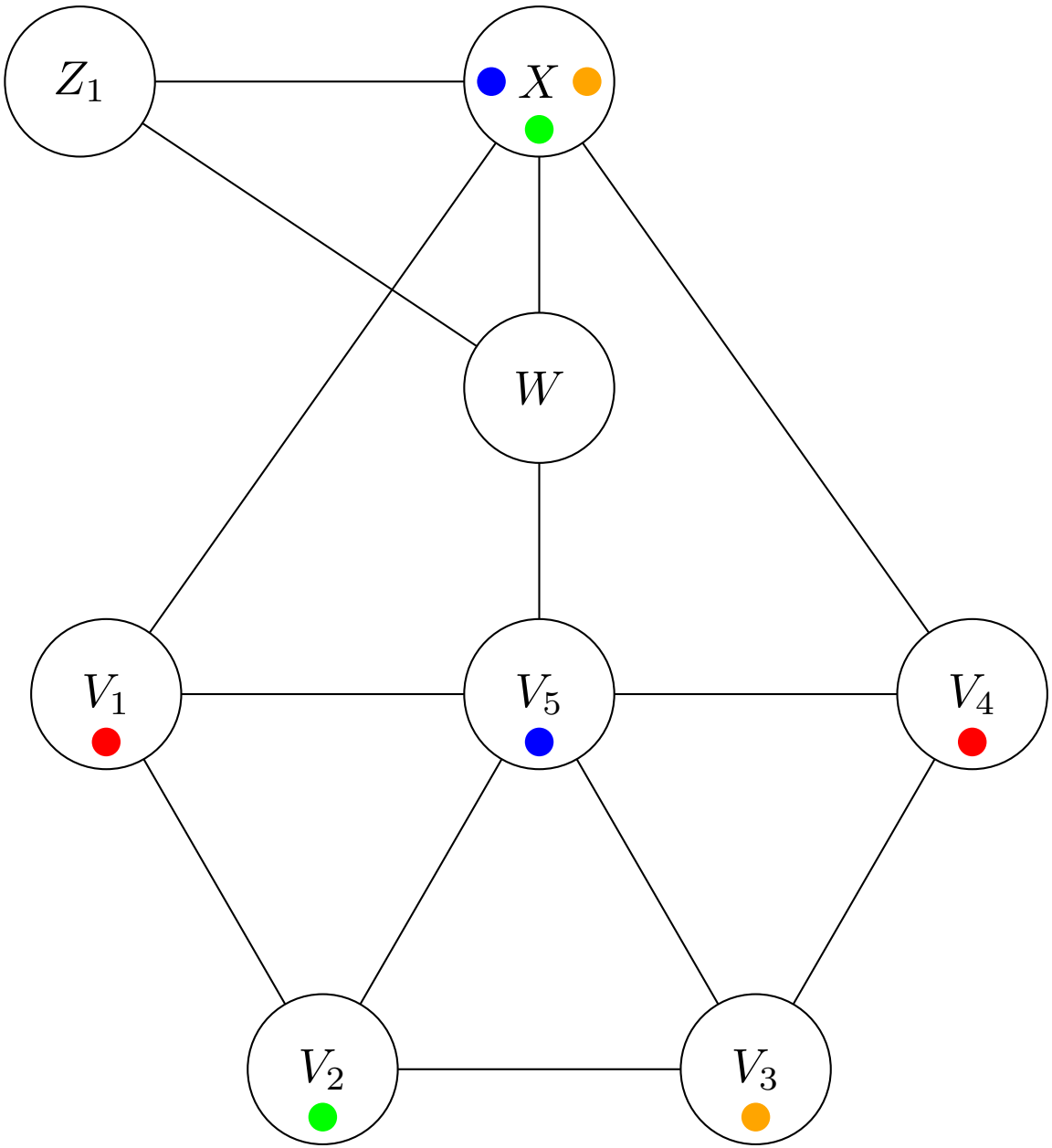
2-list-coloring



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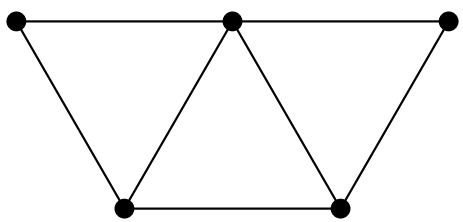
For any fixed k, there is a polynomial algorithm that determines if a (P_6, bull, gem)-free graph is k-colorable and if it is, produces a k-coloring.



gem

The List Coloring Conjecture

List-coloring

- Let G be a graph. Every vertex $v \in V(G)$ has a list $L(v)$ of prescribed colors, we want to find a proper vertex-coloring c such that $c(v) \in L(v)$.
- When such a coloring exists, G is **L-colorable**.

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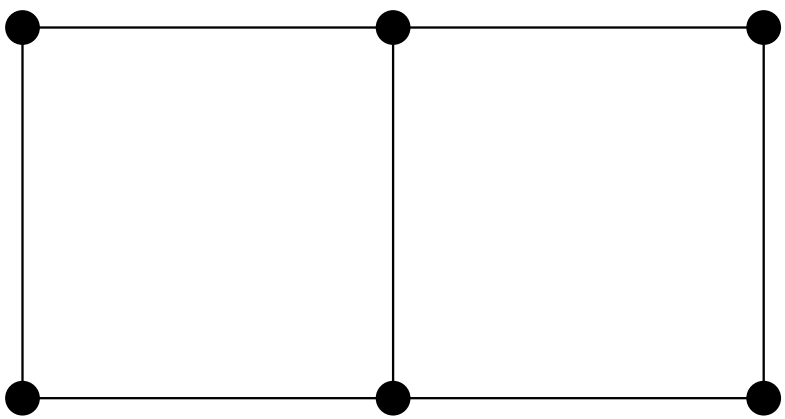

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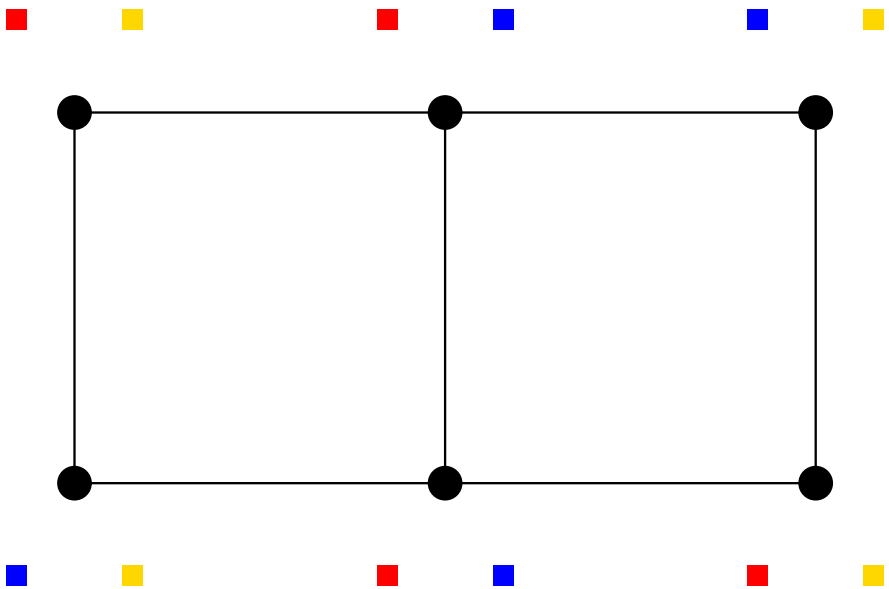

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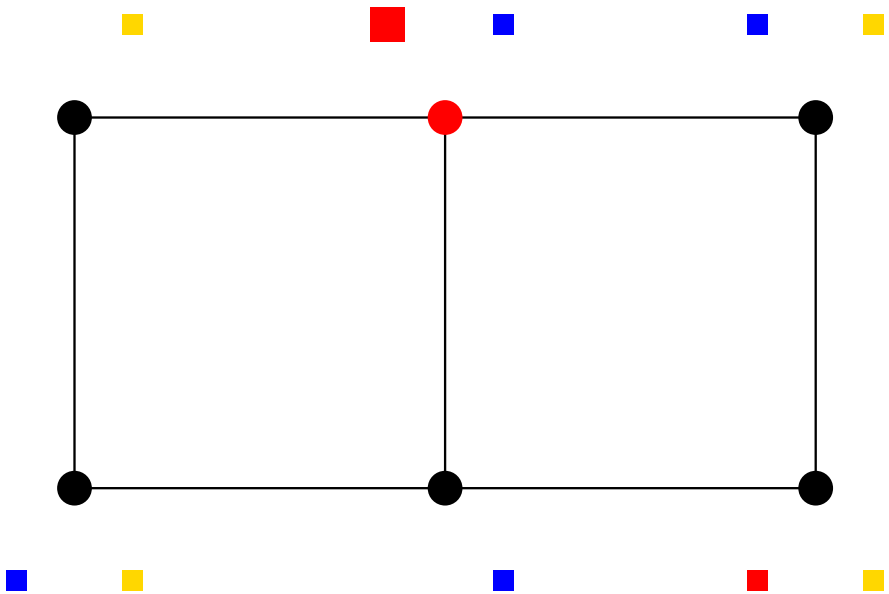

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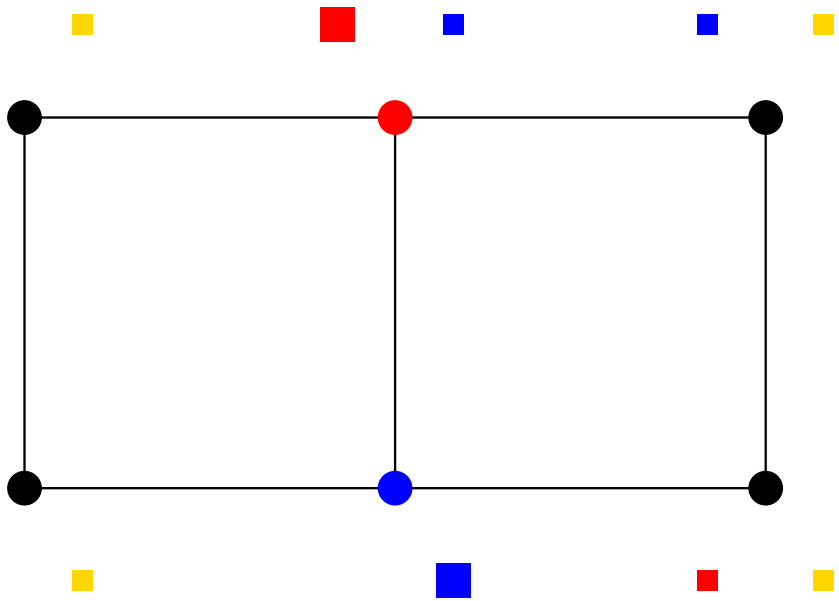

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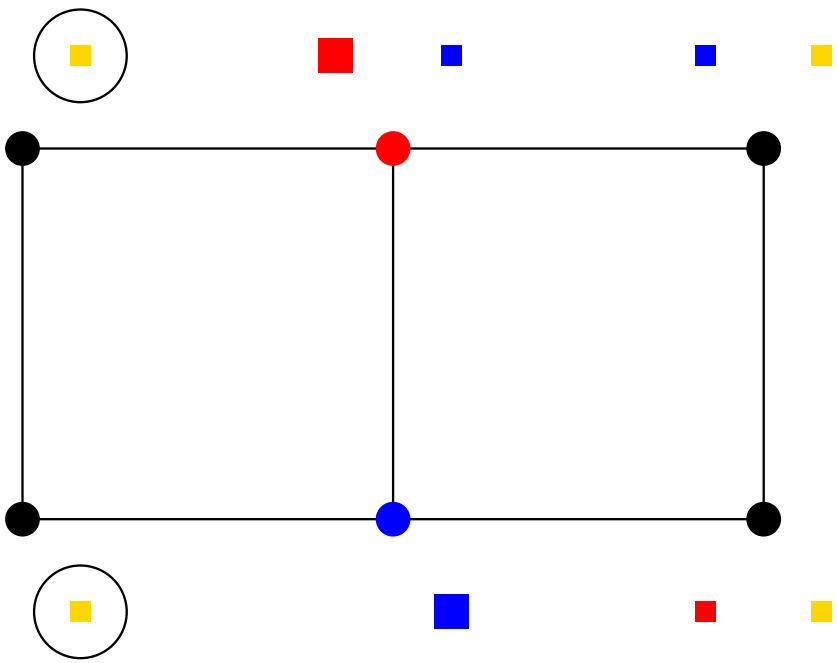

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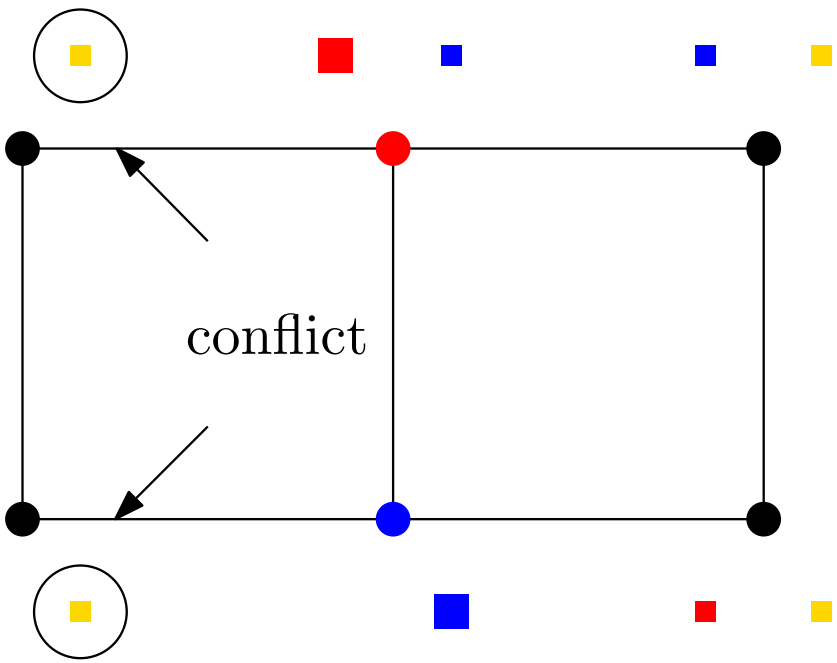

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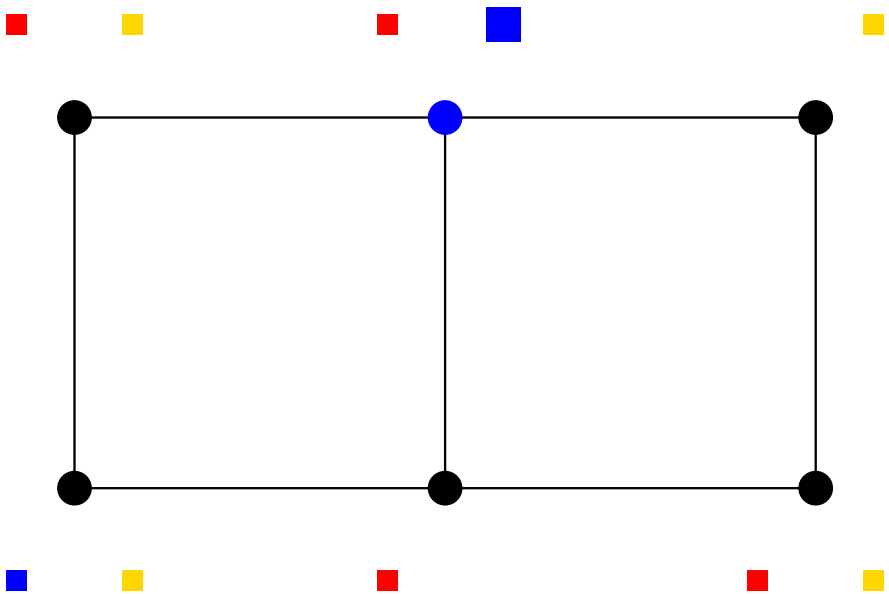

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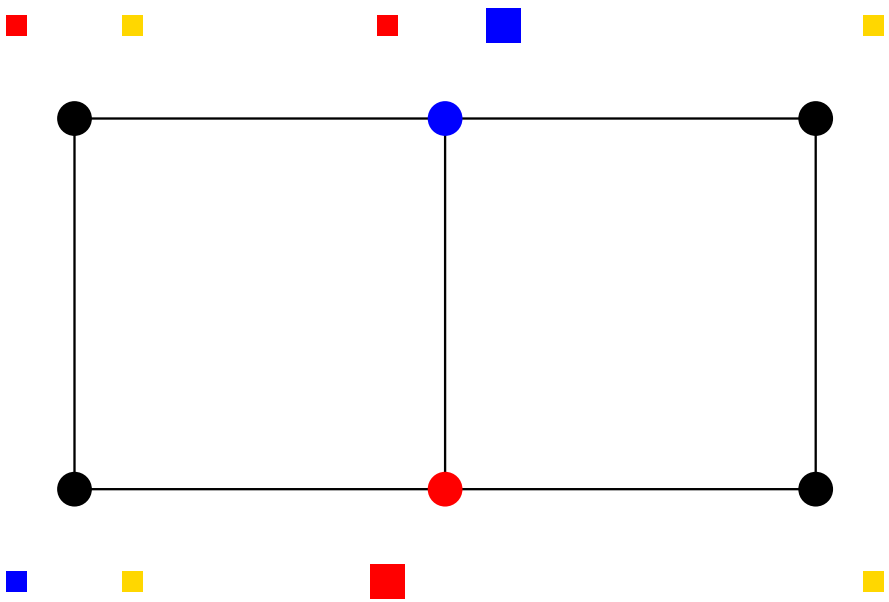

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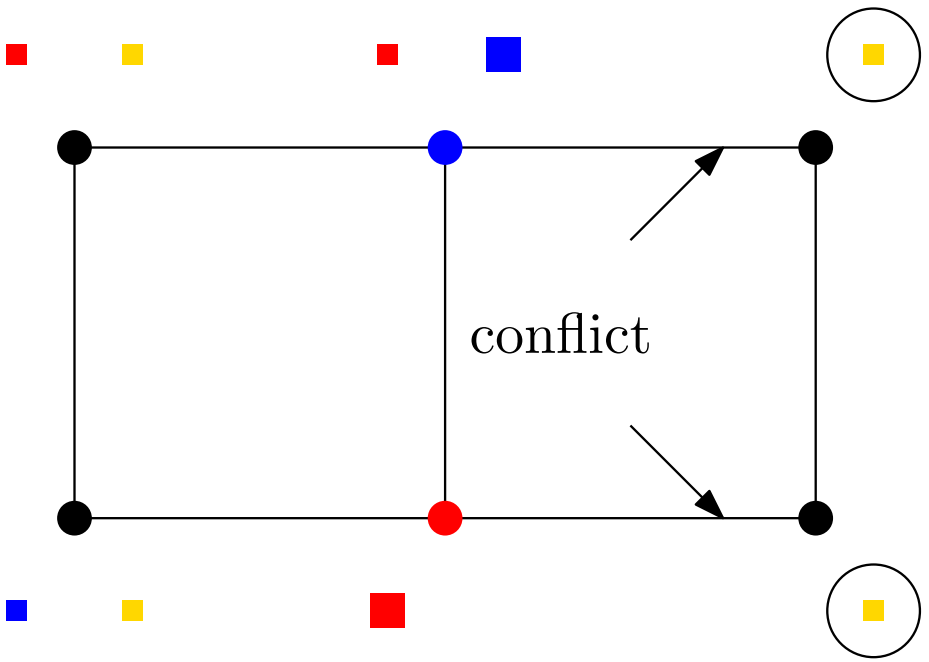

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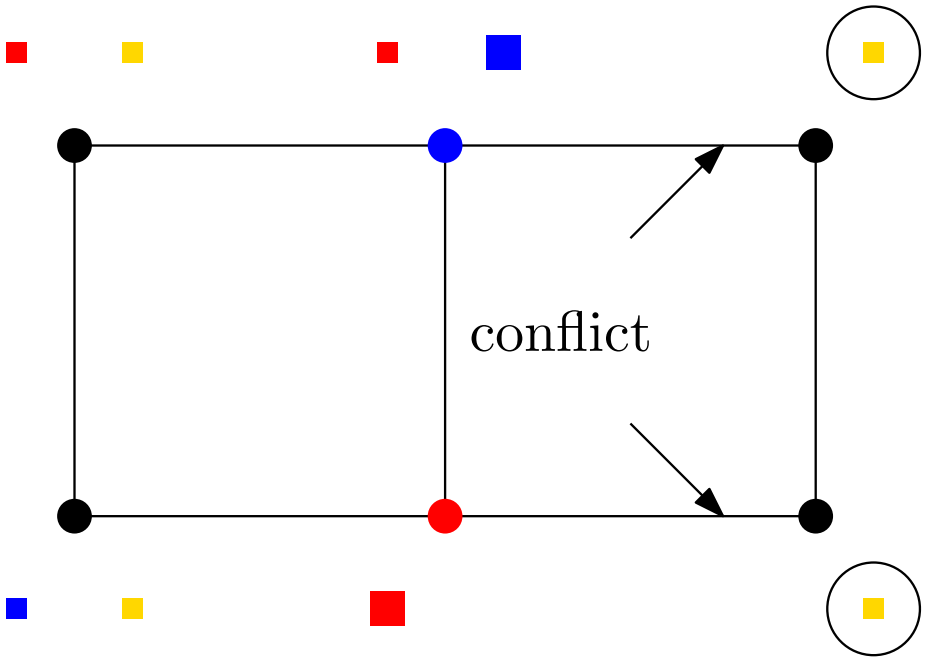

The **choice number**, denoted by $ch(G)$, is the smallest k such that for every list assignment L of size k, the graph G is L-colorable.

Vizing's conjecture 1975

For every graph G, $\chi(\mathcal{L}(G)) = ch(\mathcal{L}(G))$. In other words, $\chi'(G) = ch'(G)$.

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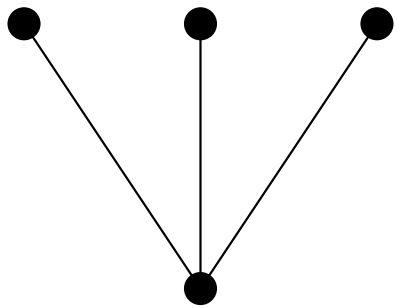
Let G be a claw-free perfect graph with $\omega(G) \leq 4$. Then $\chi(G) = ch(G)$.

Claw-free graph

The claw is the graph $K_{1,3}$. A graph is said to be claw-free if it has no induced subgraph isomorphic to $K_{1,3}$.



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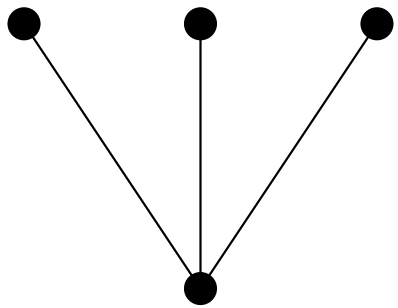
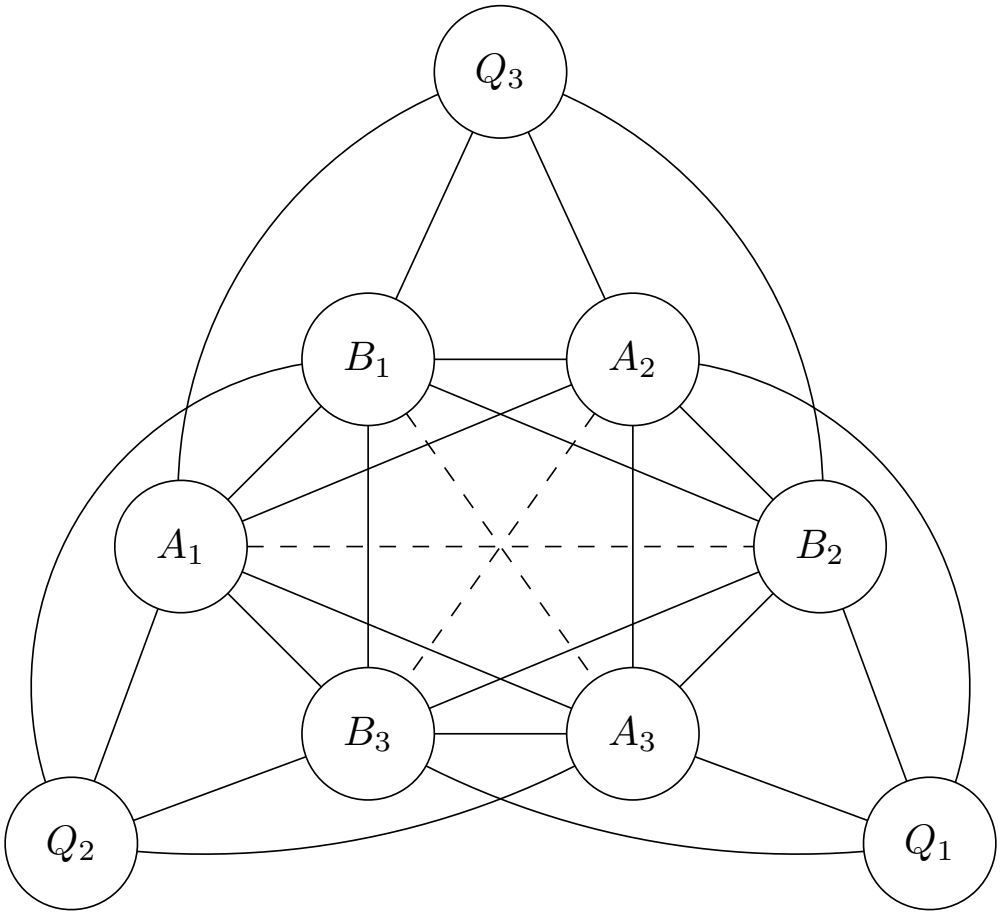


claw

Theorem [*Chvátal, Sbihi 1988*]

Every claw-free perfect graph either has a **clique-cutset**, or is a peculiar graph, or is an elementary graph.

peculiar



clique

at least one non-edge

complete adjacency

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- If no such pair exists, we can find a coloring by Hall's theorem.

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A graph G is **elementary** if and only if it is an **augmentation** of the line-graph H of a bipartite multigraph B.

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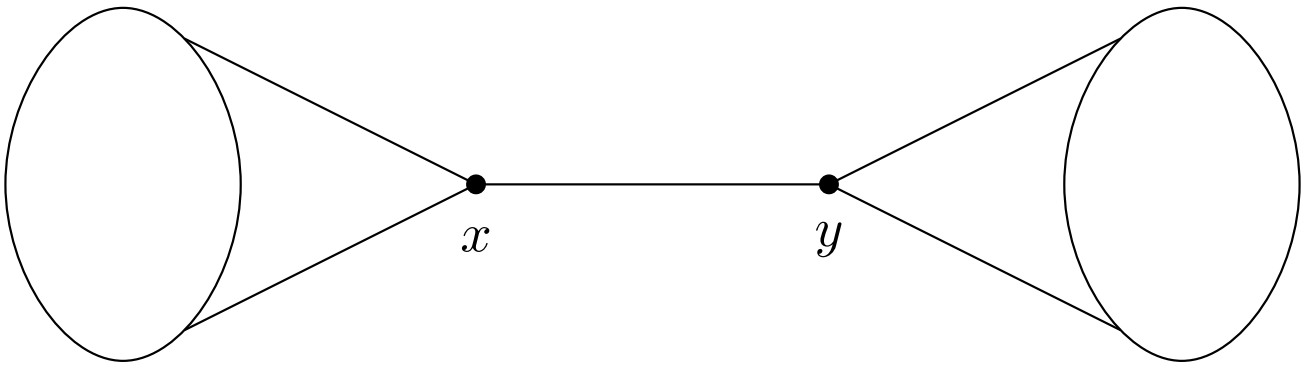

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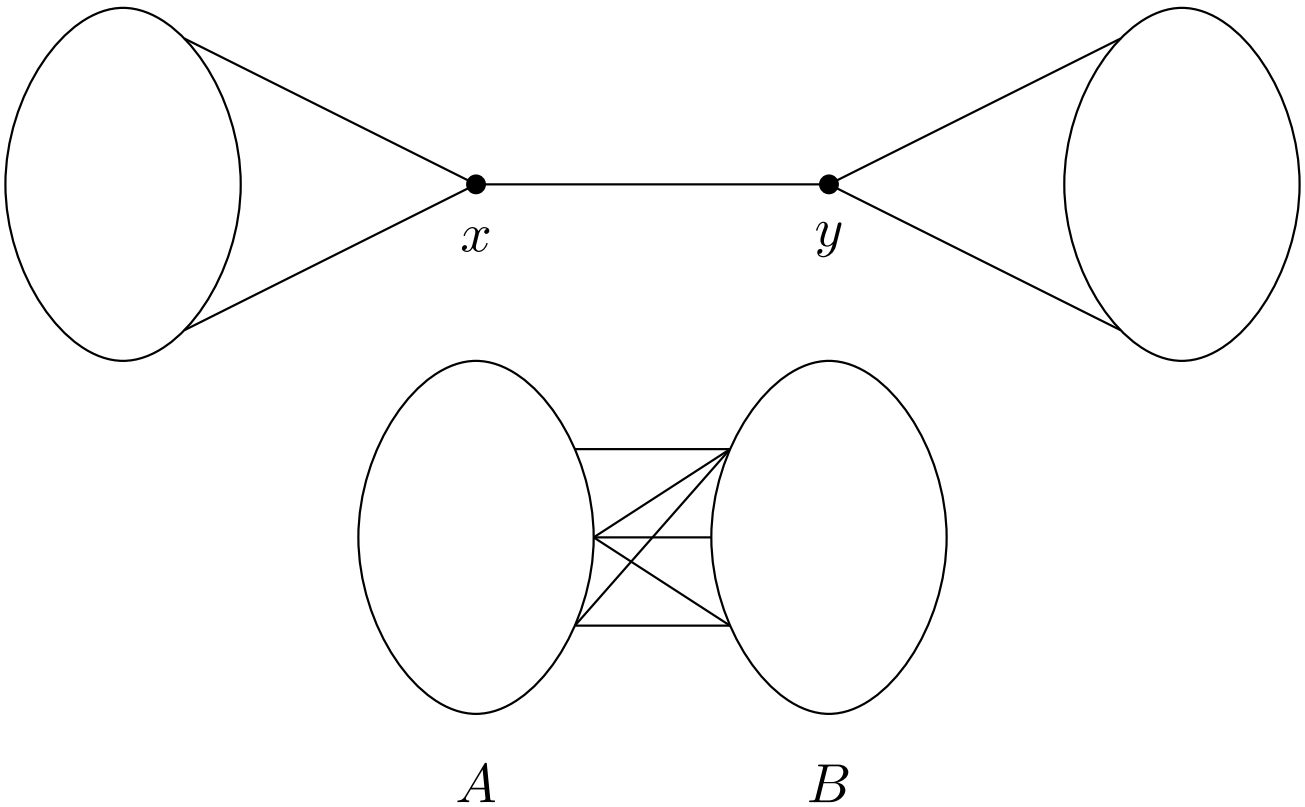

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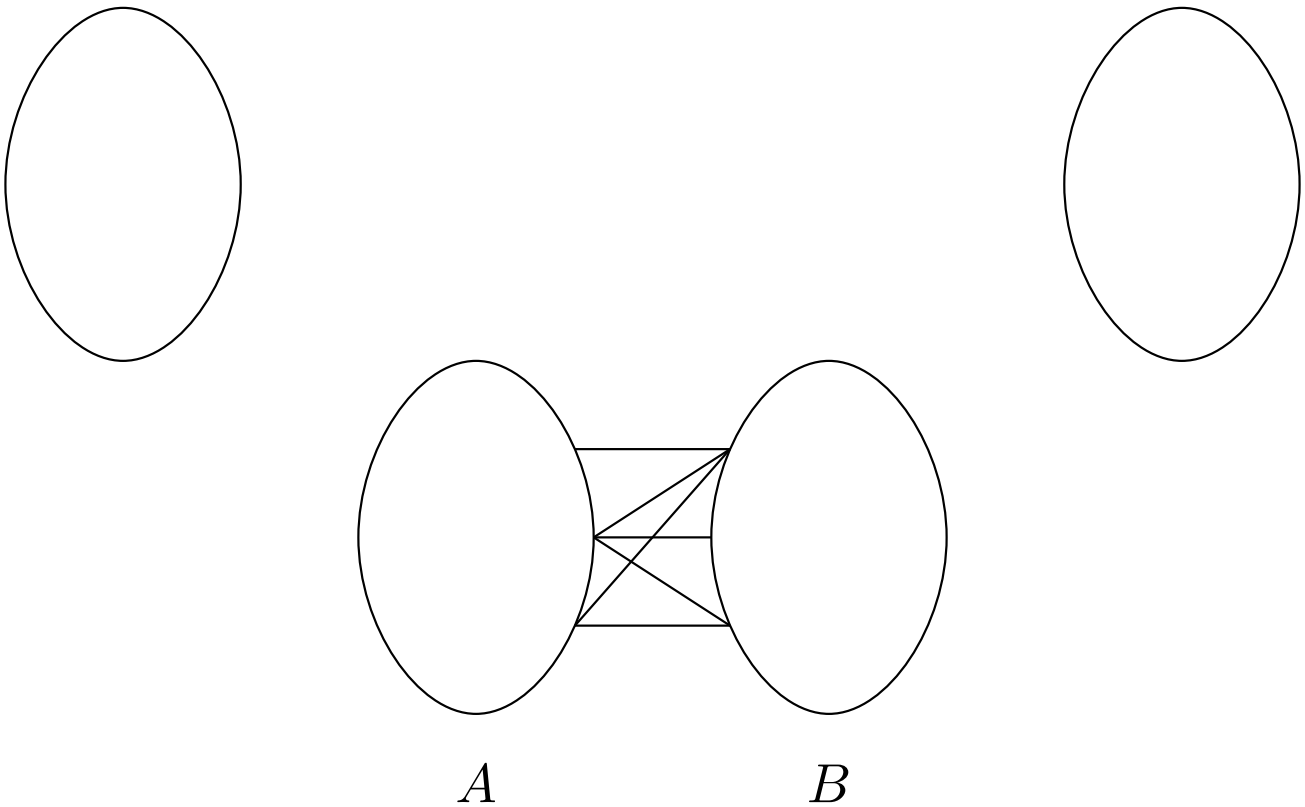

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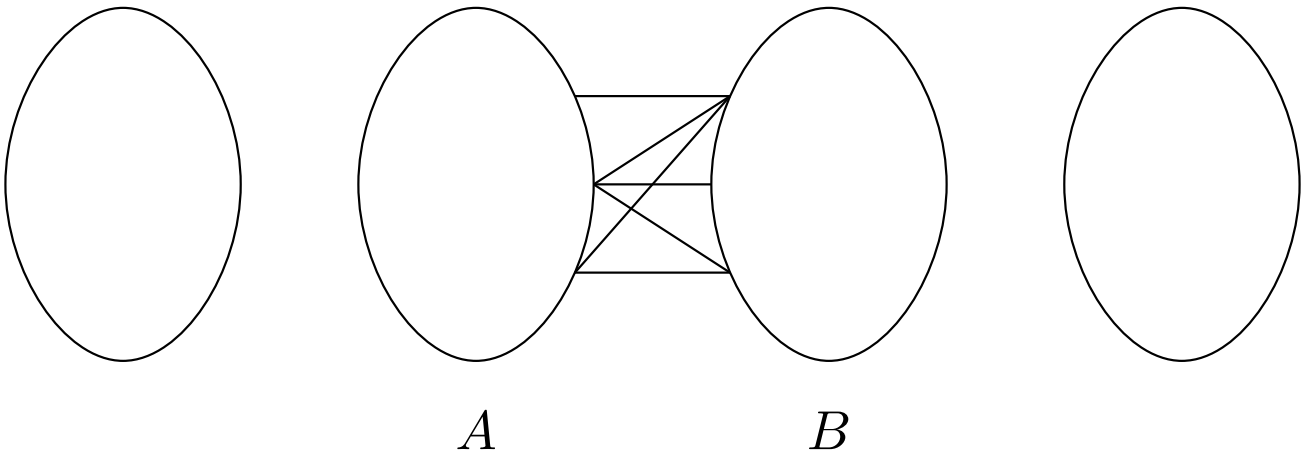

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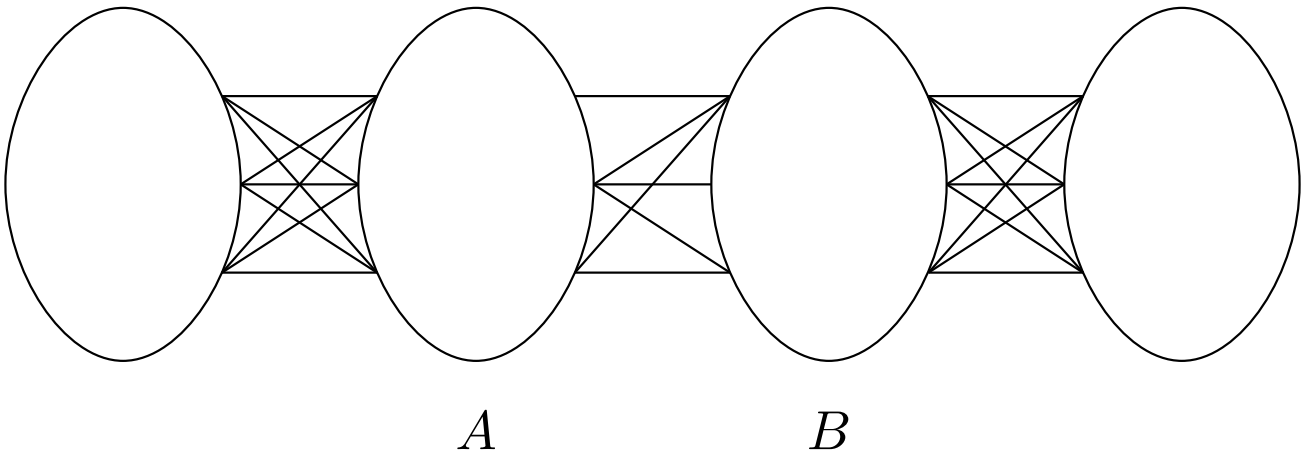

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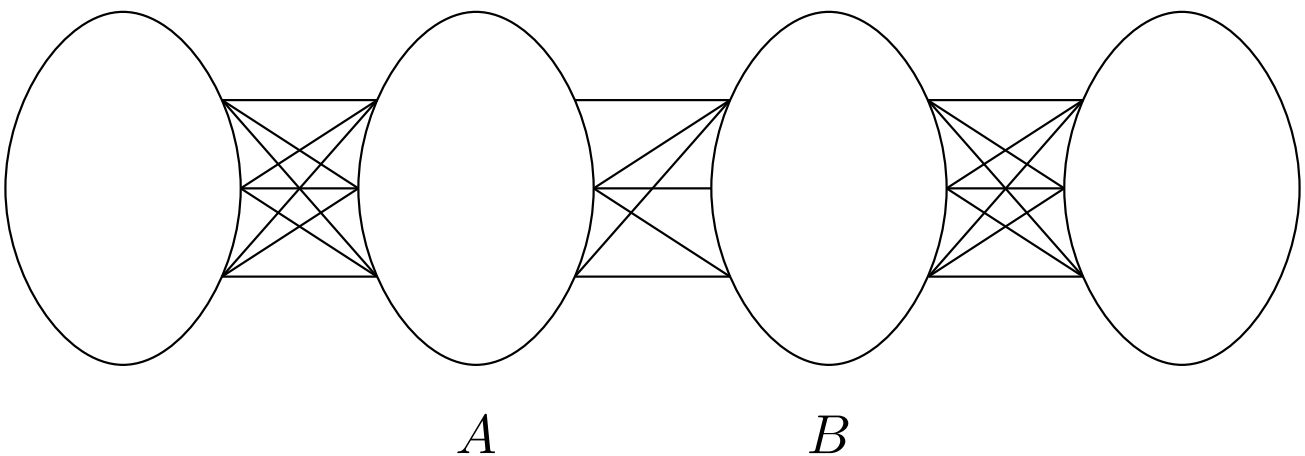

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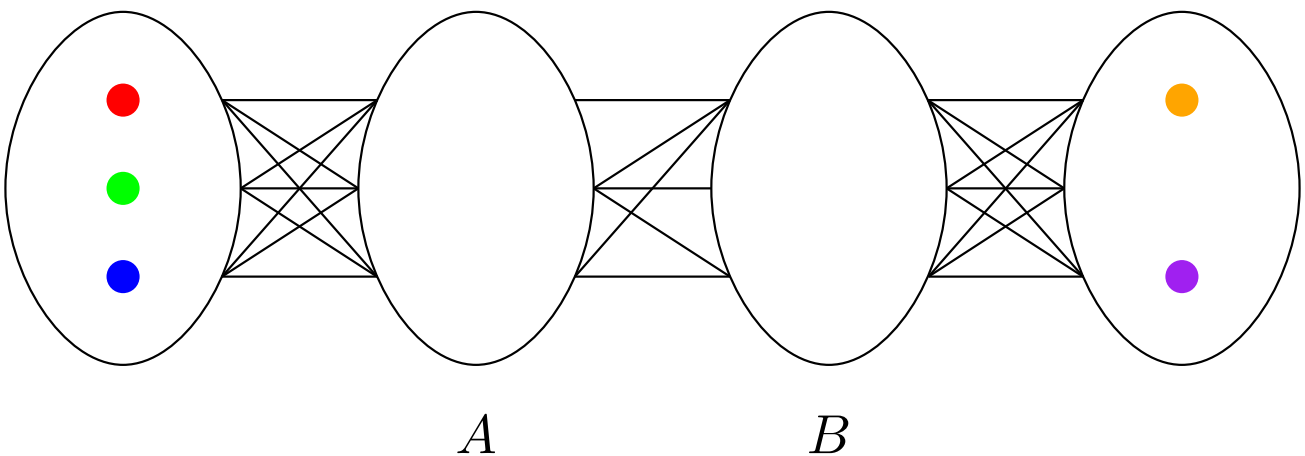

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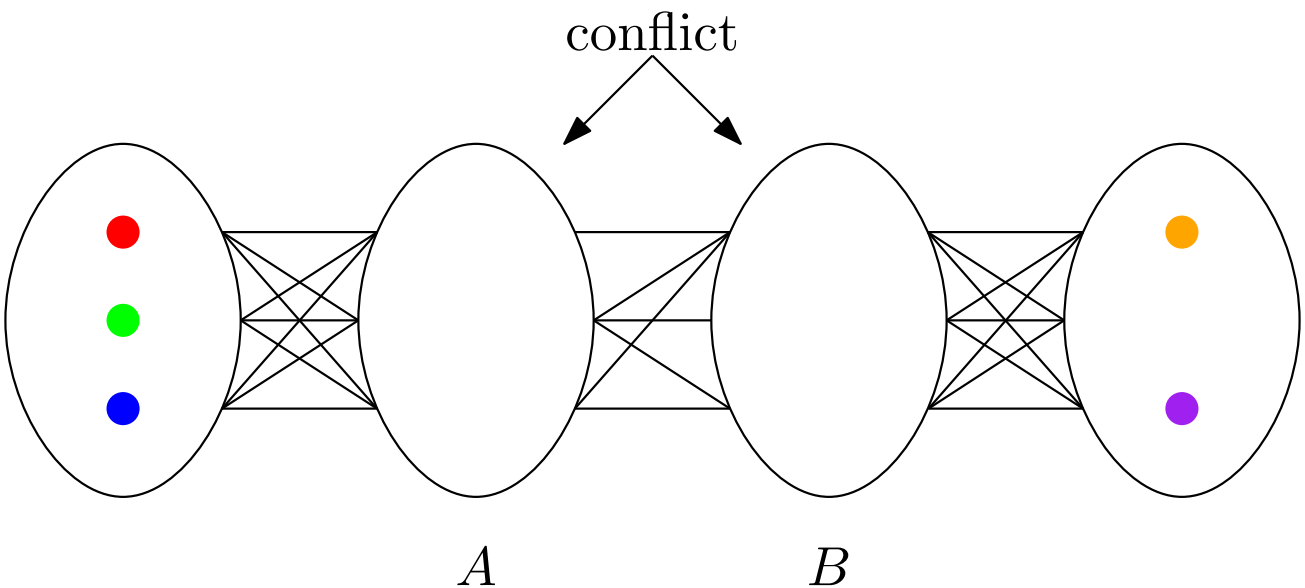

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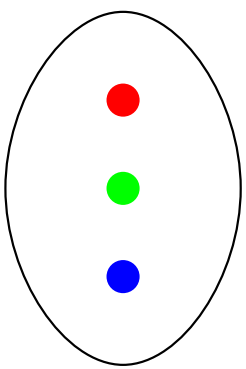

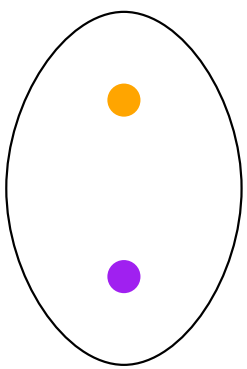

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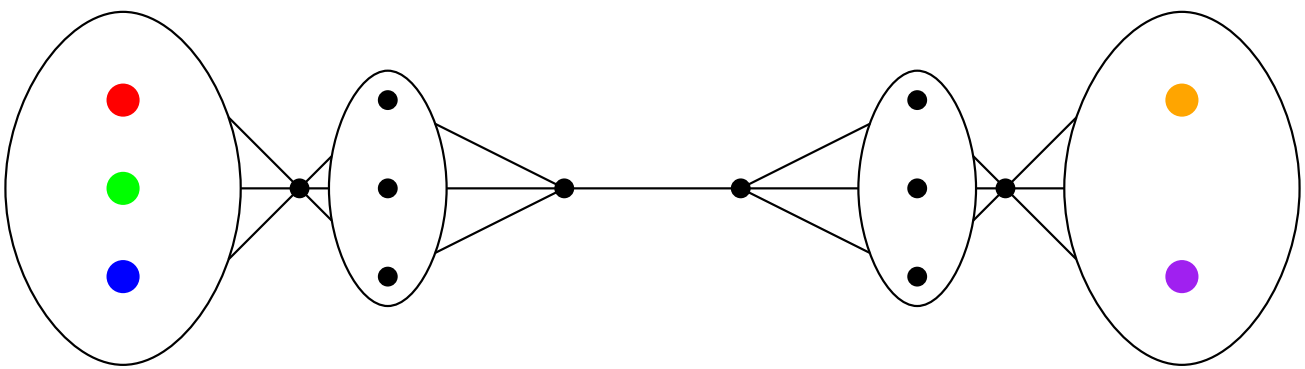

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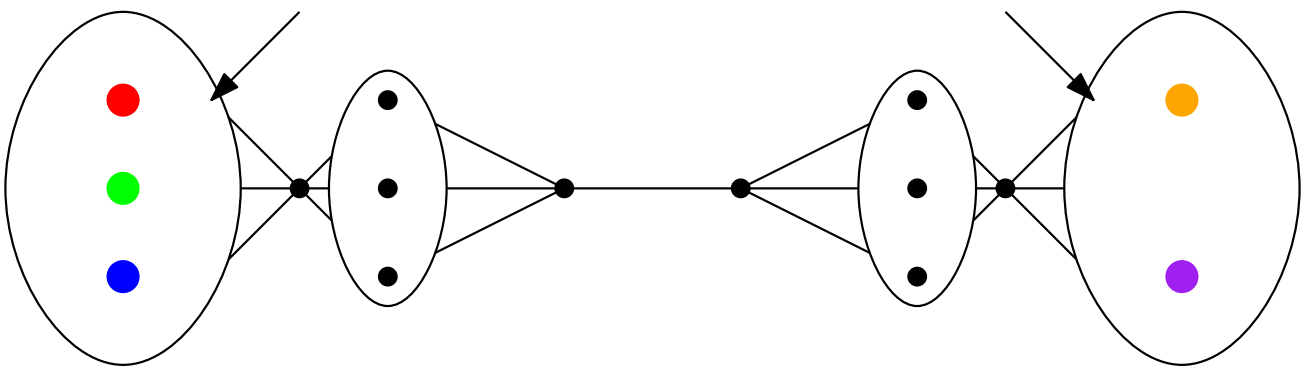

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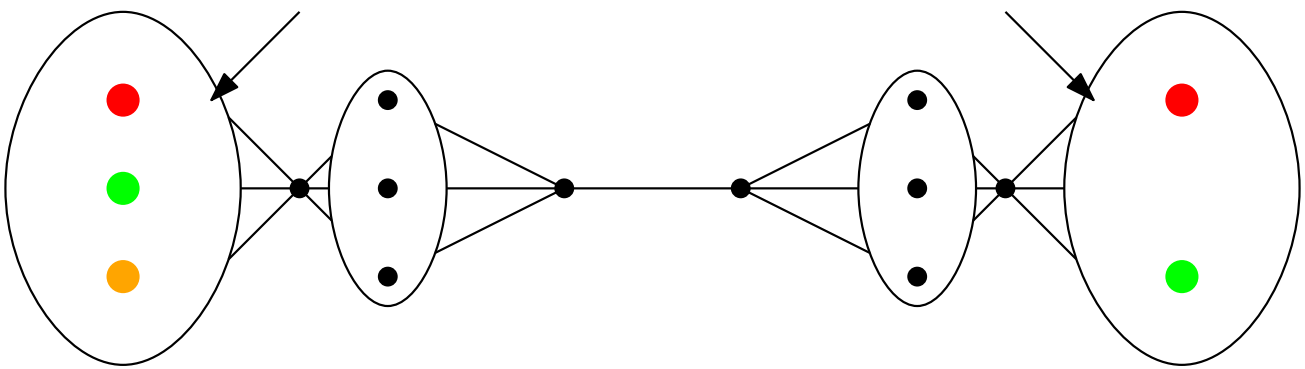

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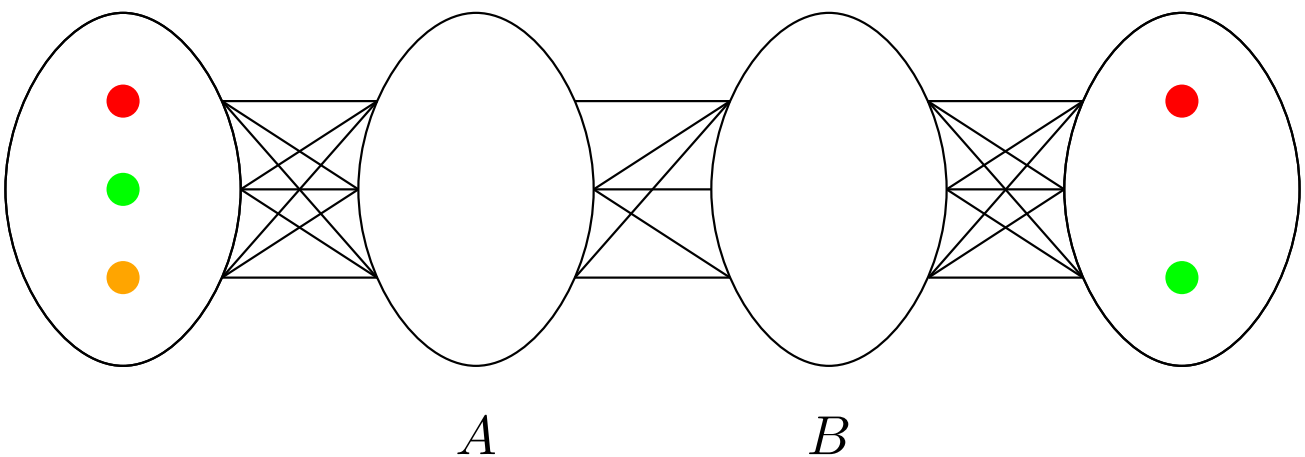

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Within the list-coloring context, clique cutsets are not as convenient as for the classical coloring.
We still manage to deal with them by using **Galvin's theorem.**

Theorem [*Gravier, Maffray, Pastor*]

Let G be a claw-free perfect graph with $\omega(G) \leq 4$. Then $\chi(G) = ch(G)$.

The Normal Graph Conjecture

Definition

A graph G is **normal** if there exists a covering of $V(G)$, $\mathbb{C}$, of cliques and a covering of $V(G)$, $\mathbb{S}$, of stable sets such that $C \cap S \neq \emptyset$ for every $C \in \mathbb{C}$ and $S \in \mathbb{S}$.

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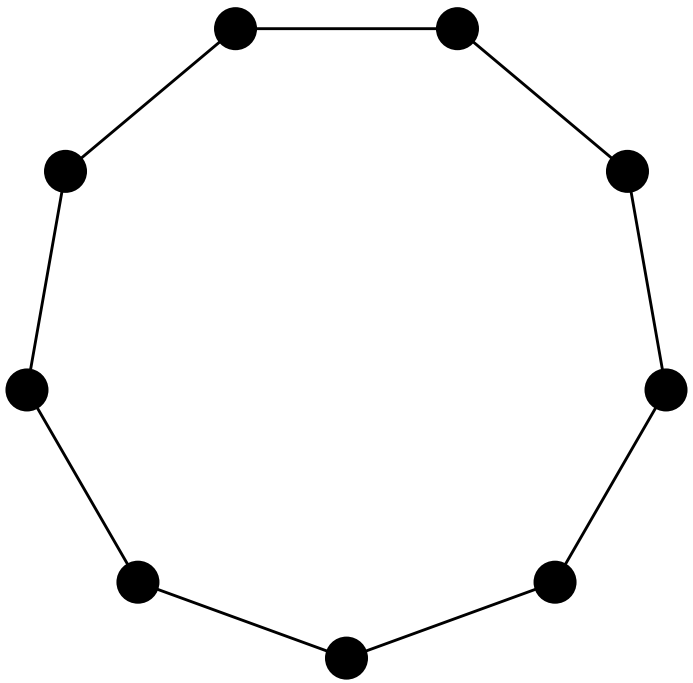

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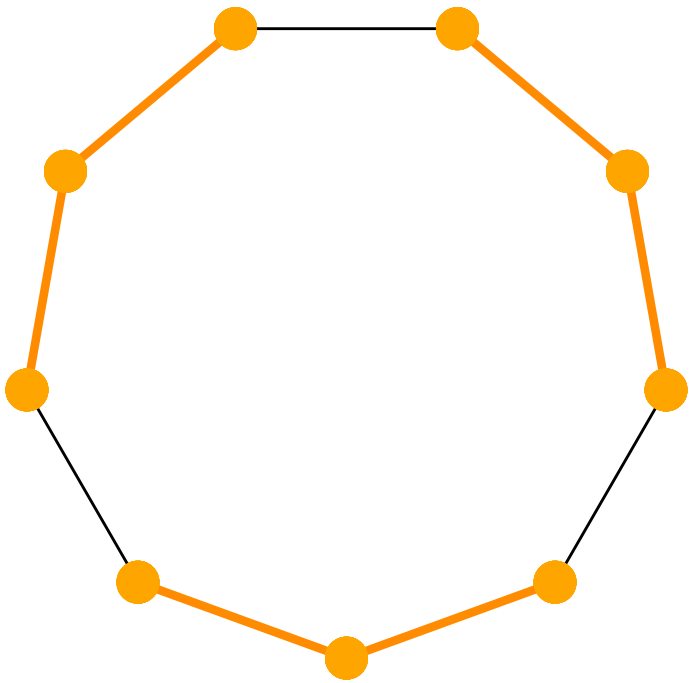

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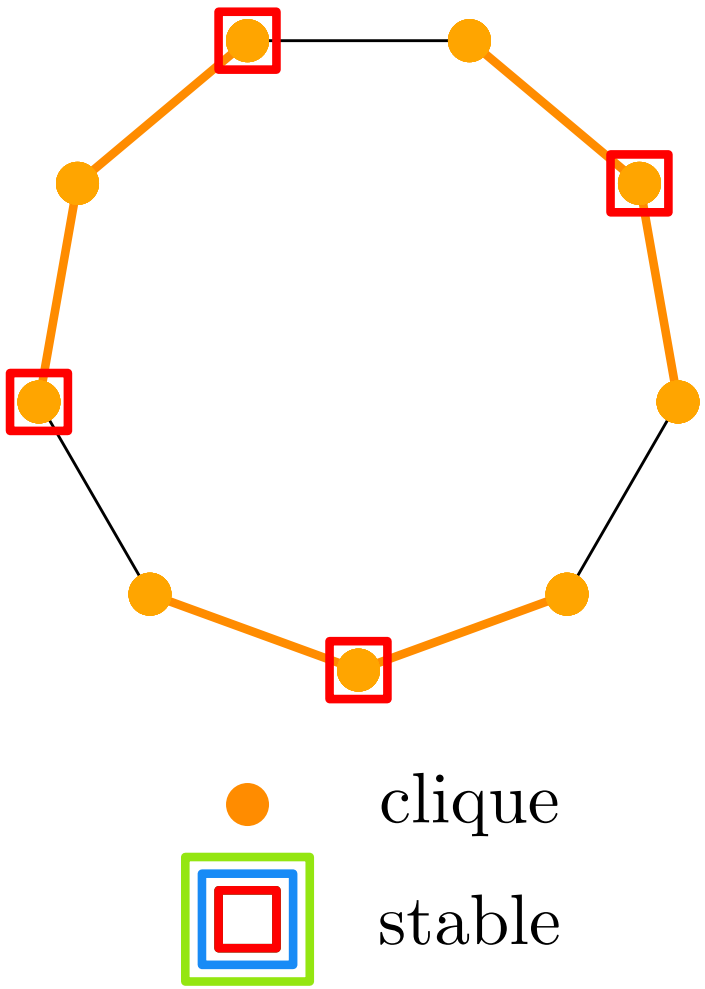

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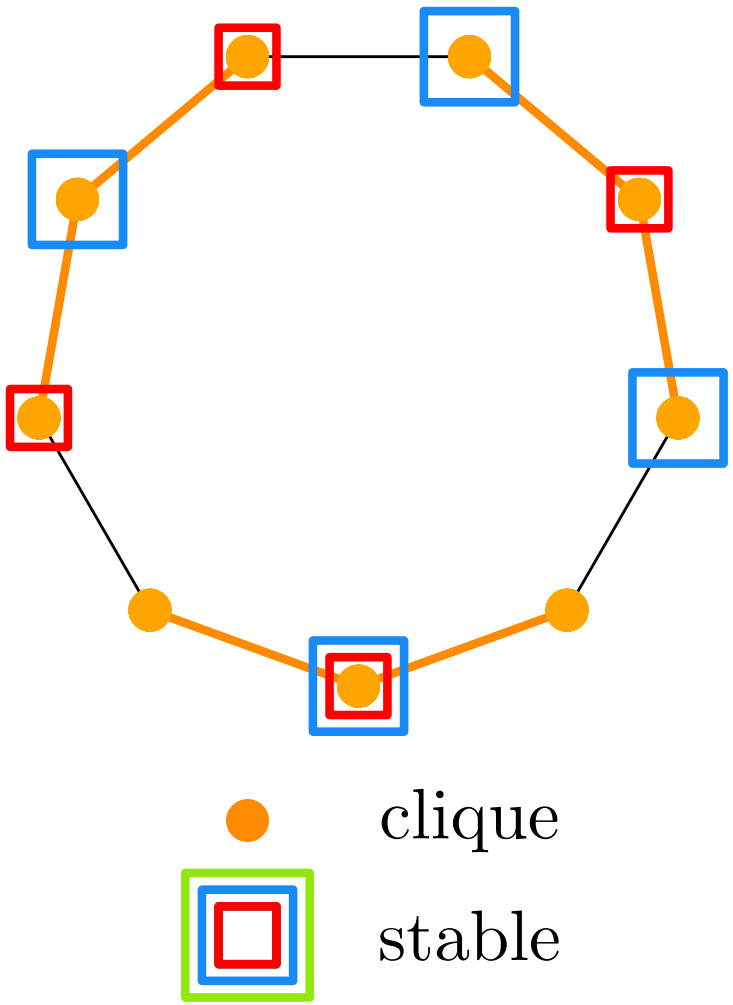

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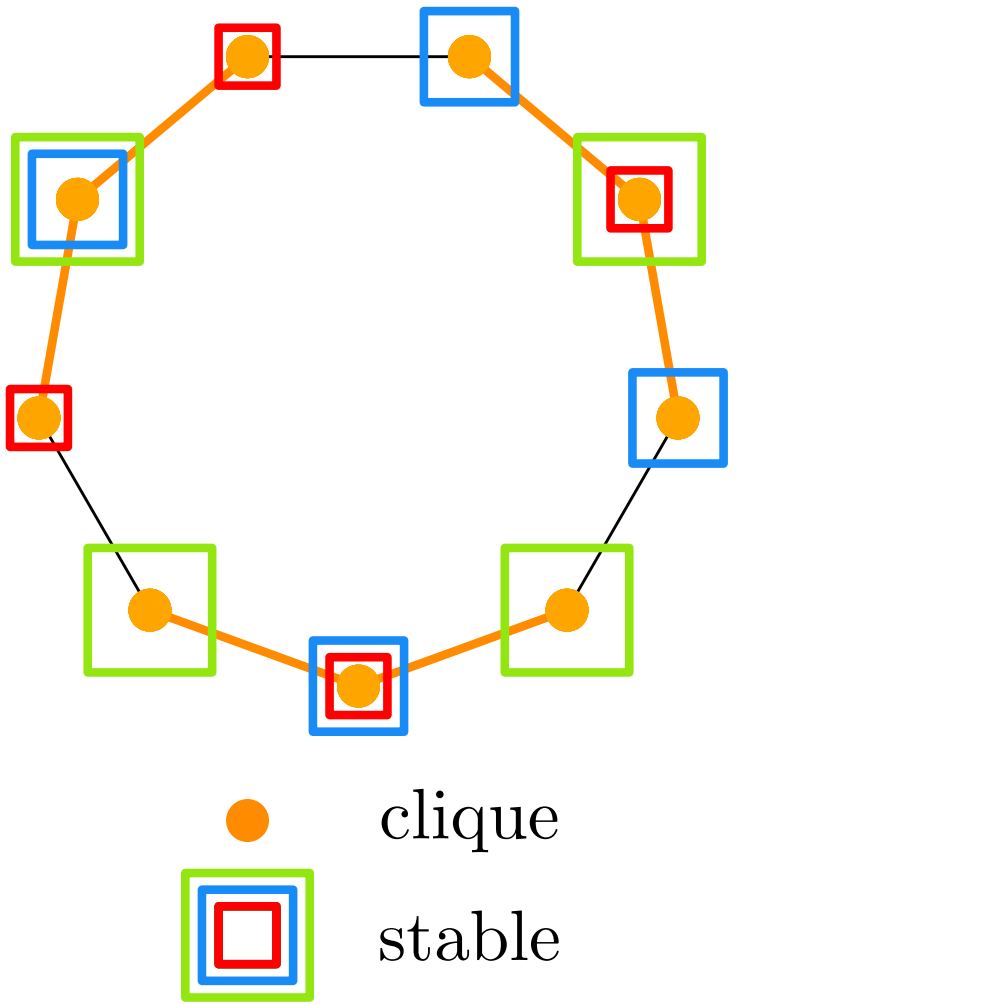

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- Almost all d-regular graphs are normal when d is fixed [*Hosseini, Mohar, Rezaei 2015*].

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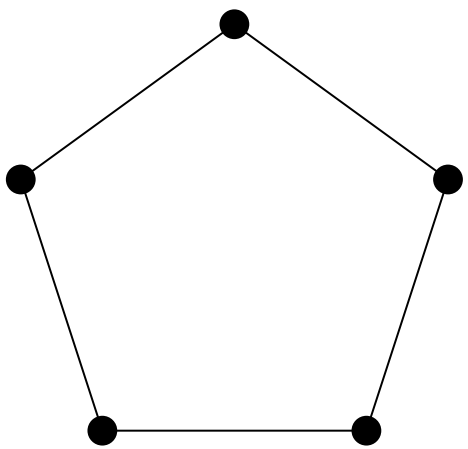

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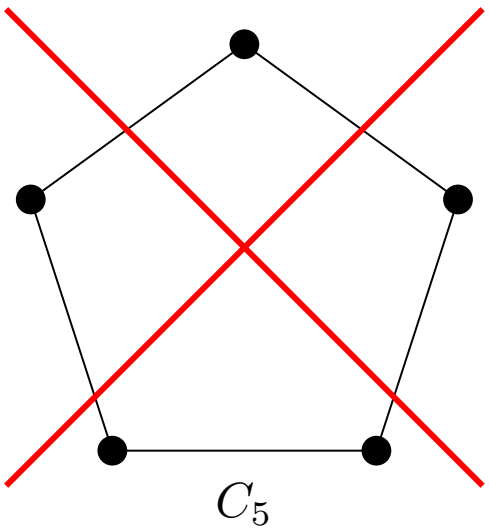

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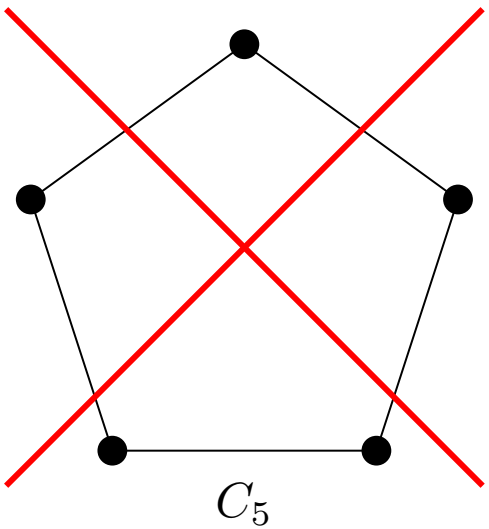

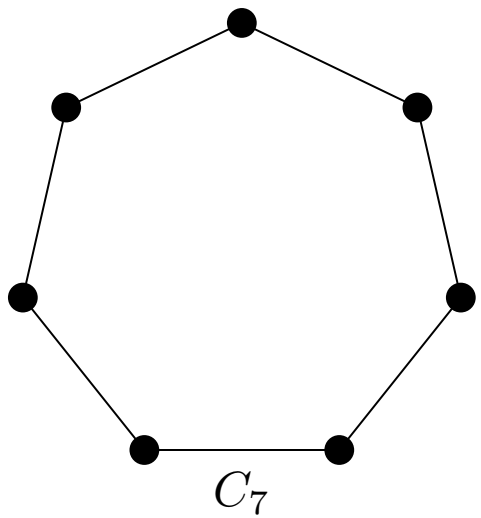

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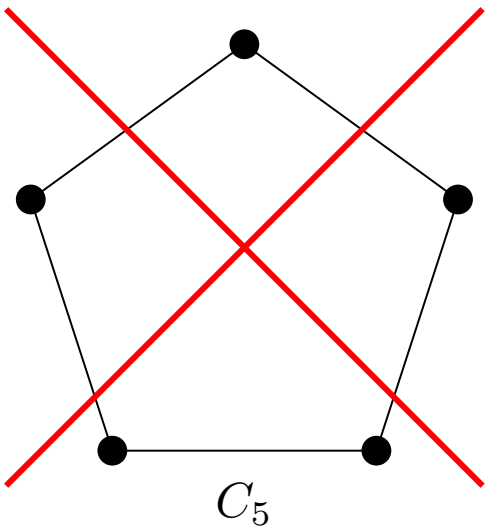

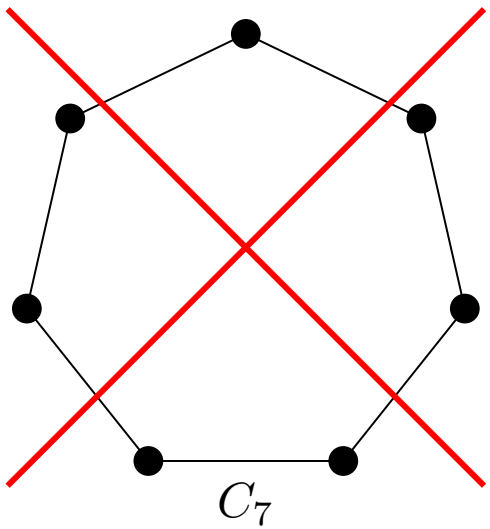

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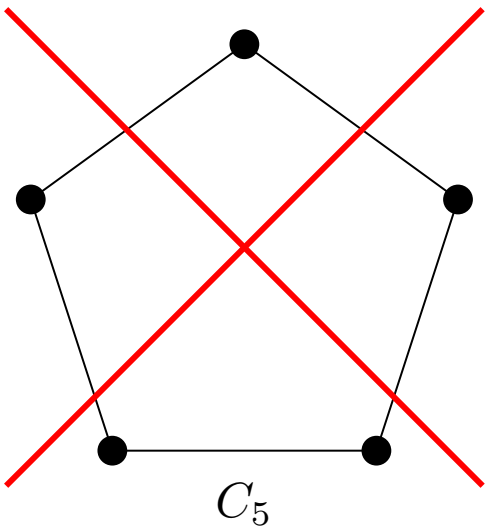

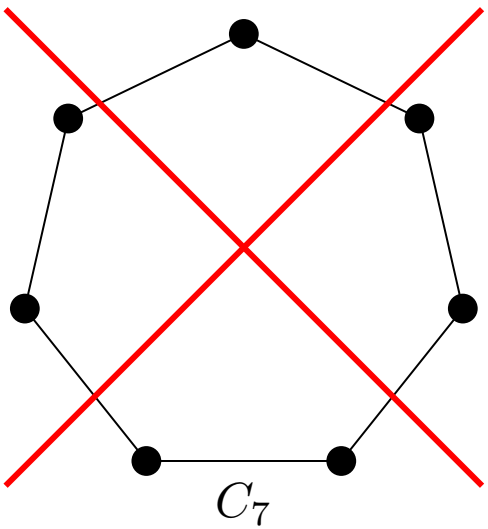

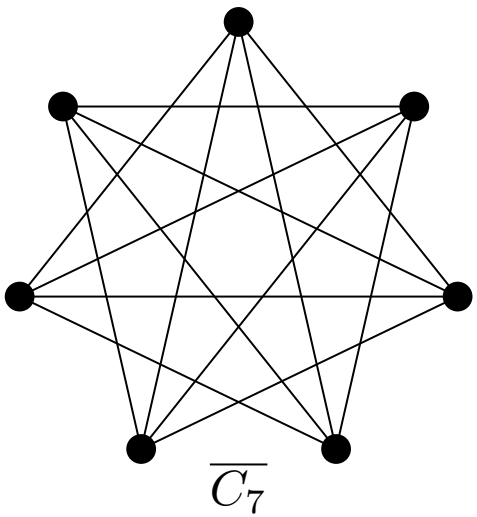

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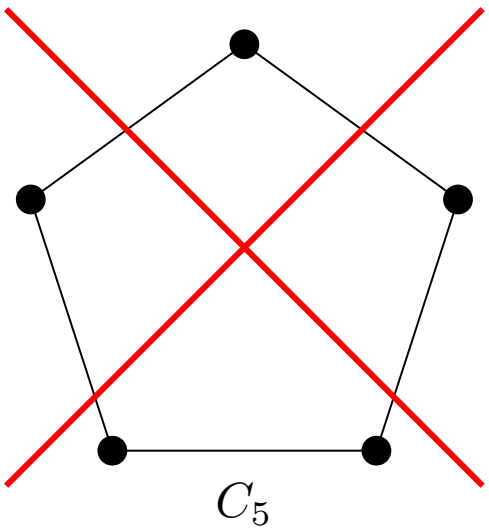

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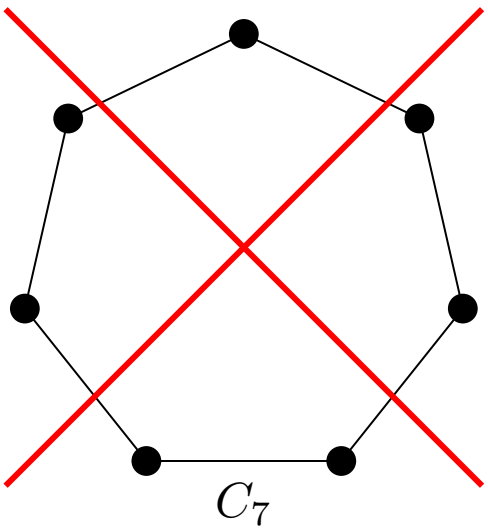

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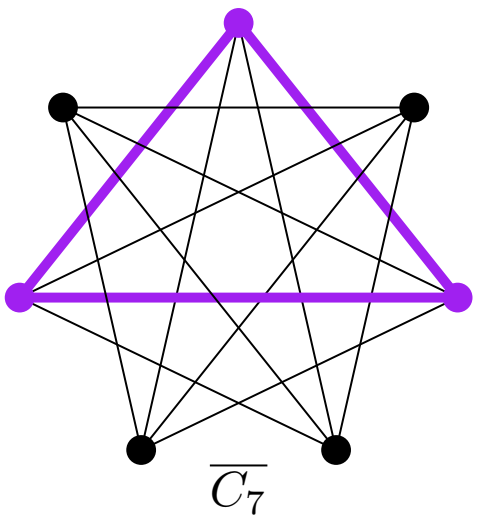

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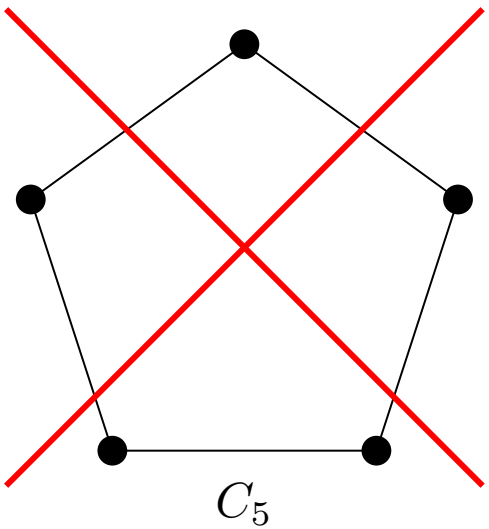

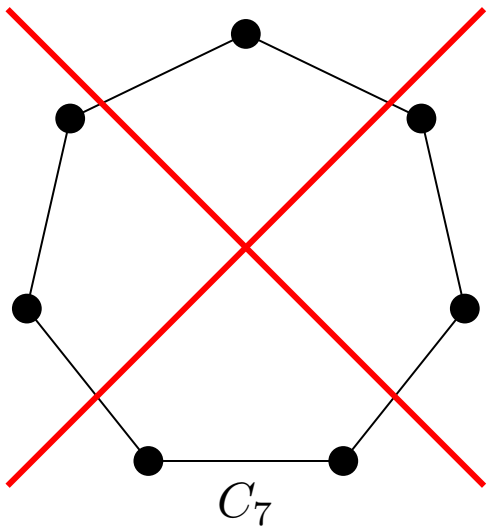

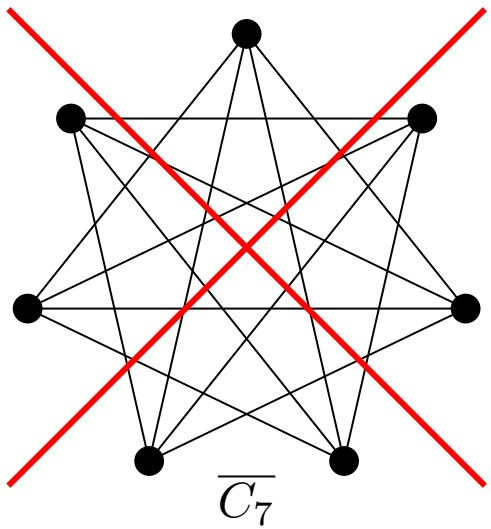

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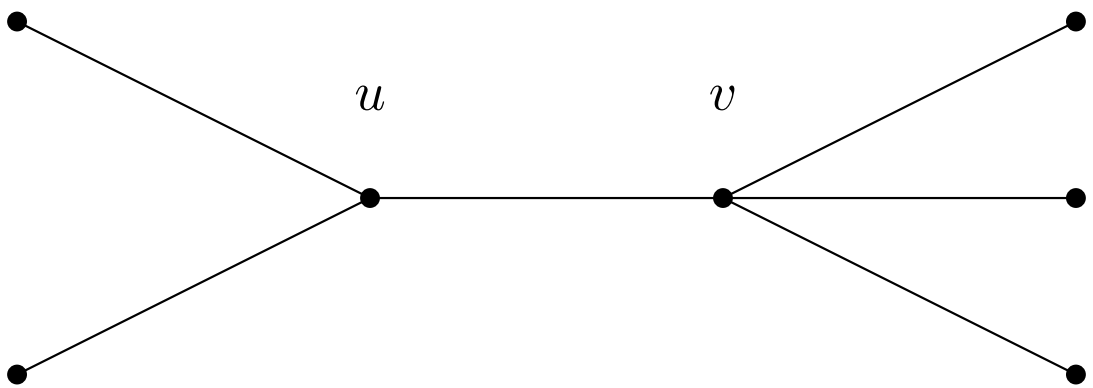

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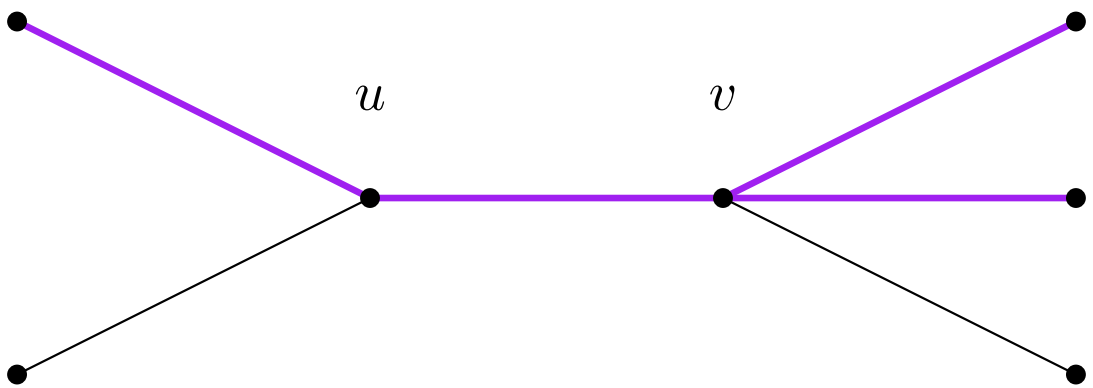

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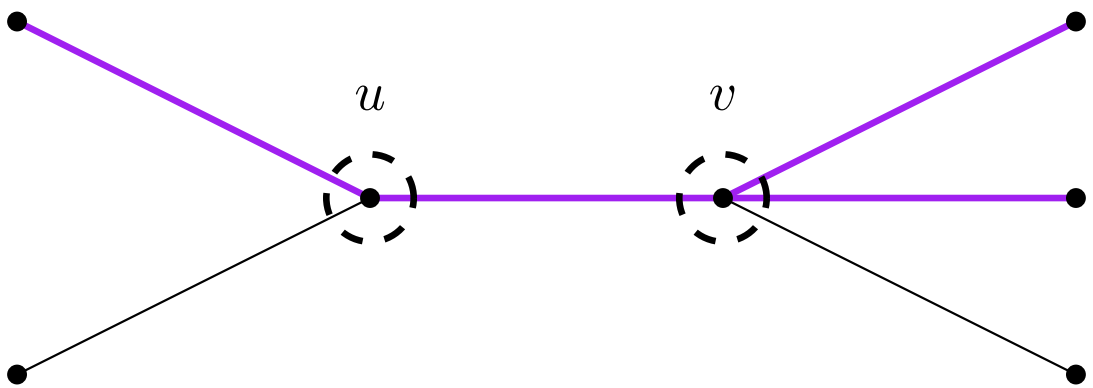

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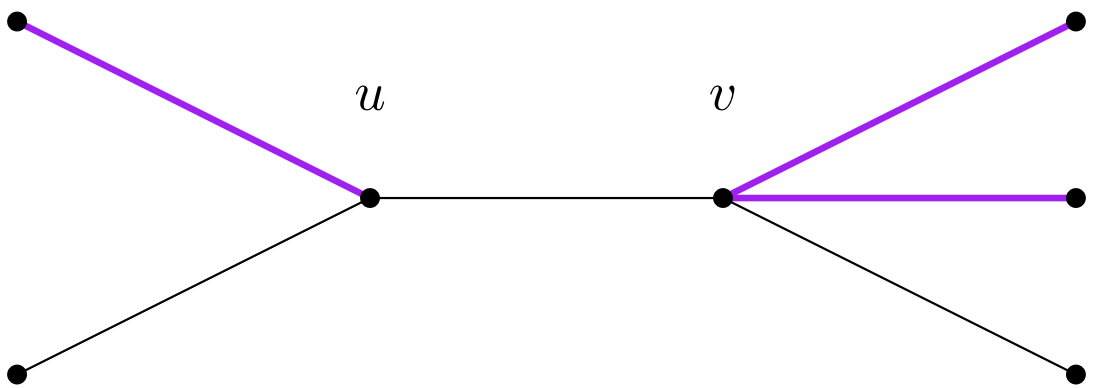

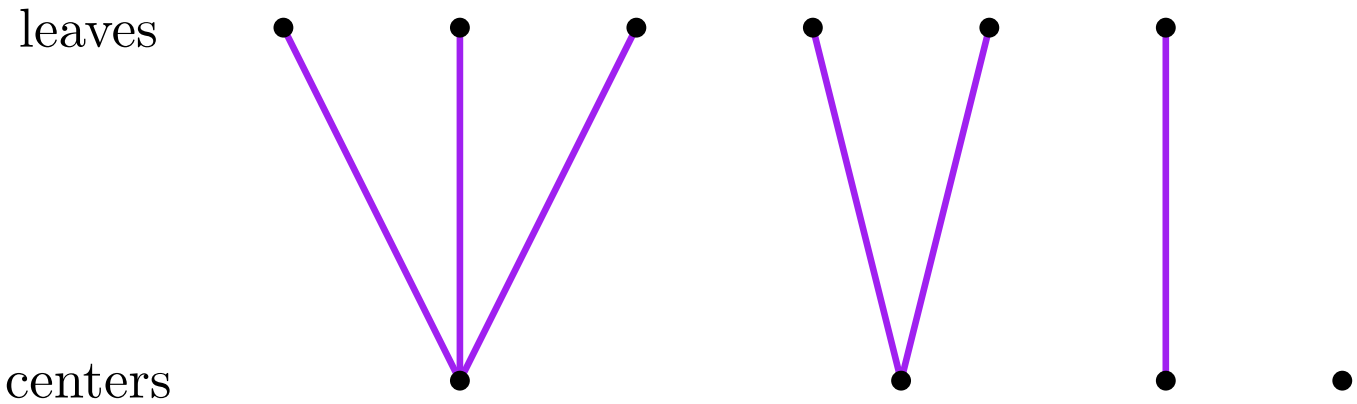
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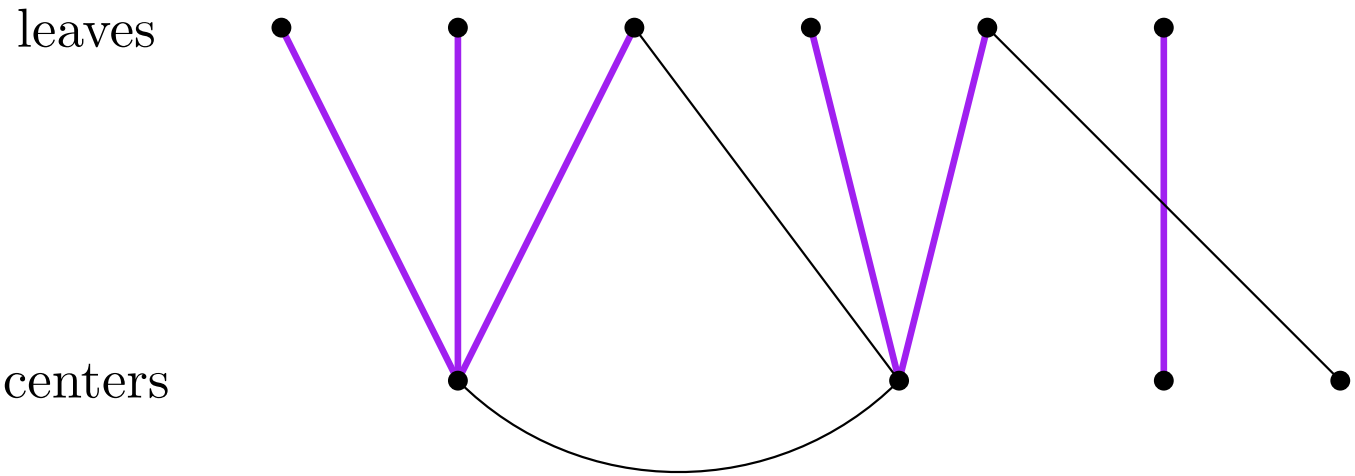
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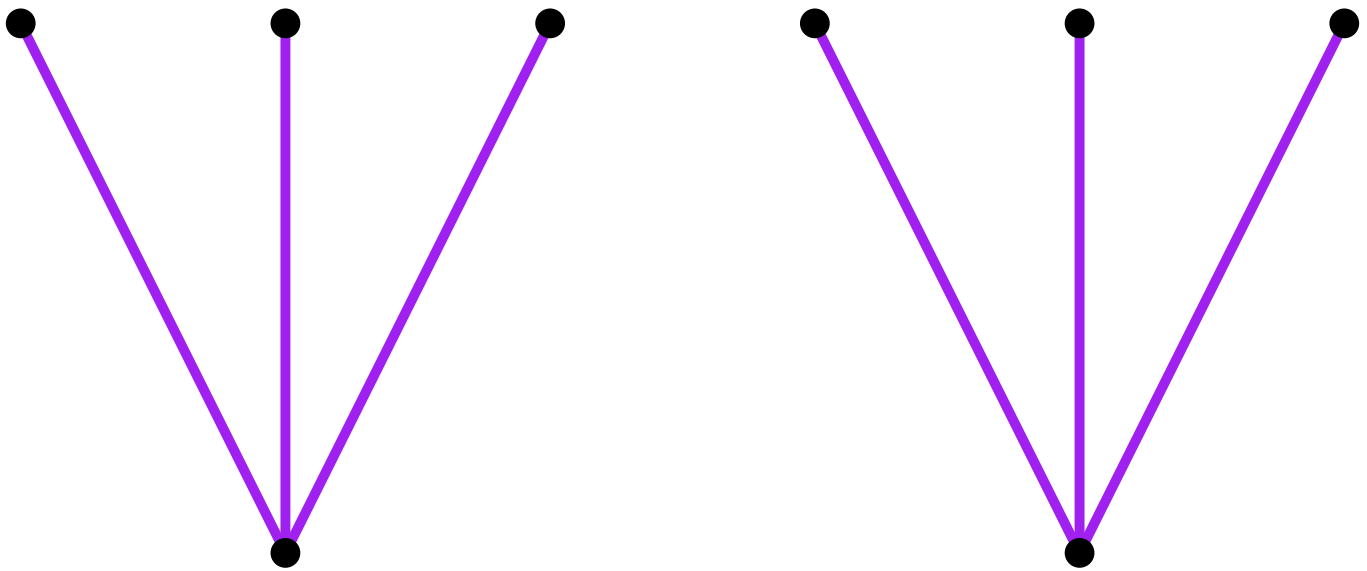

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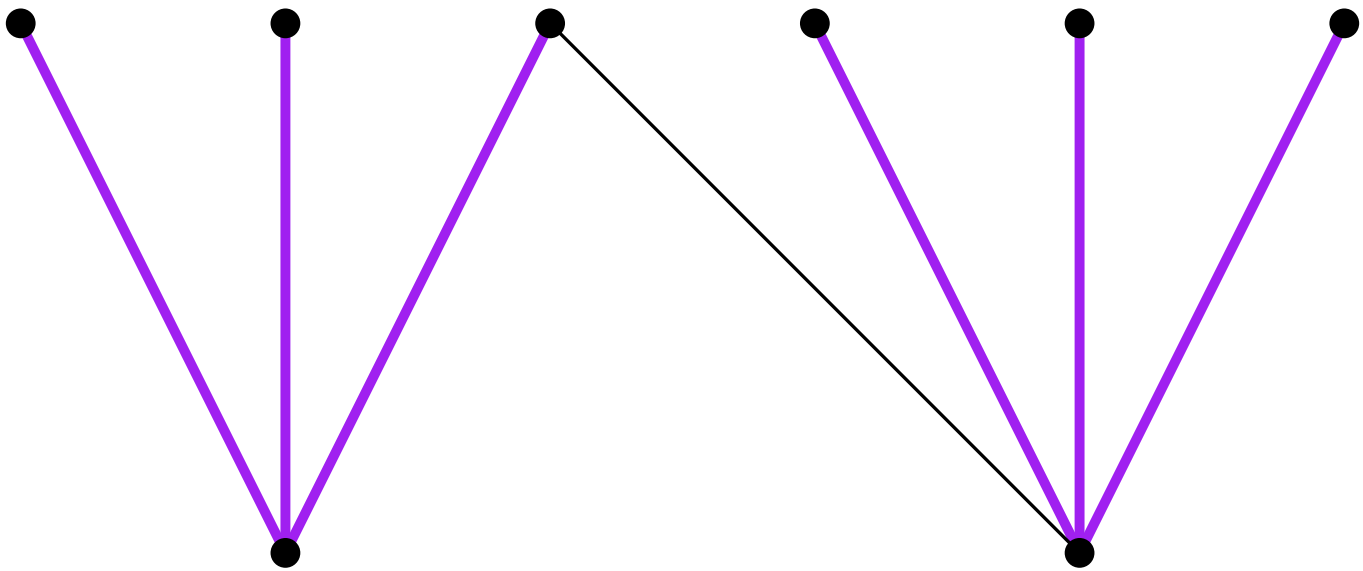

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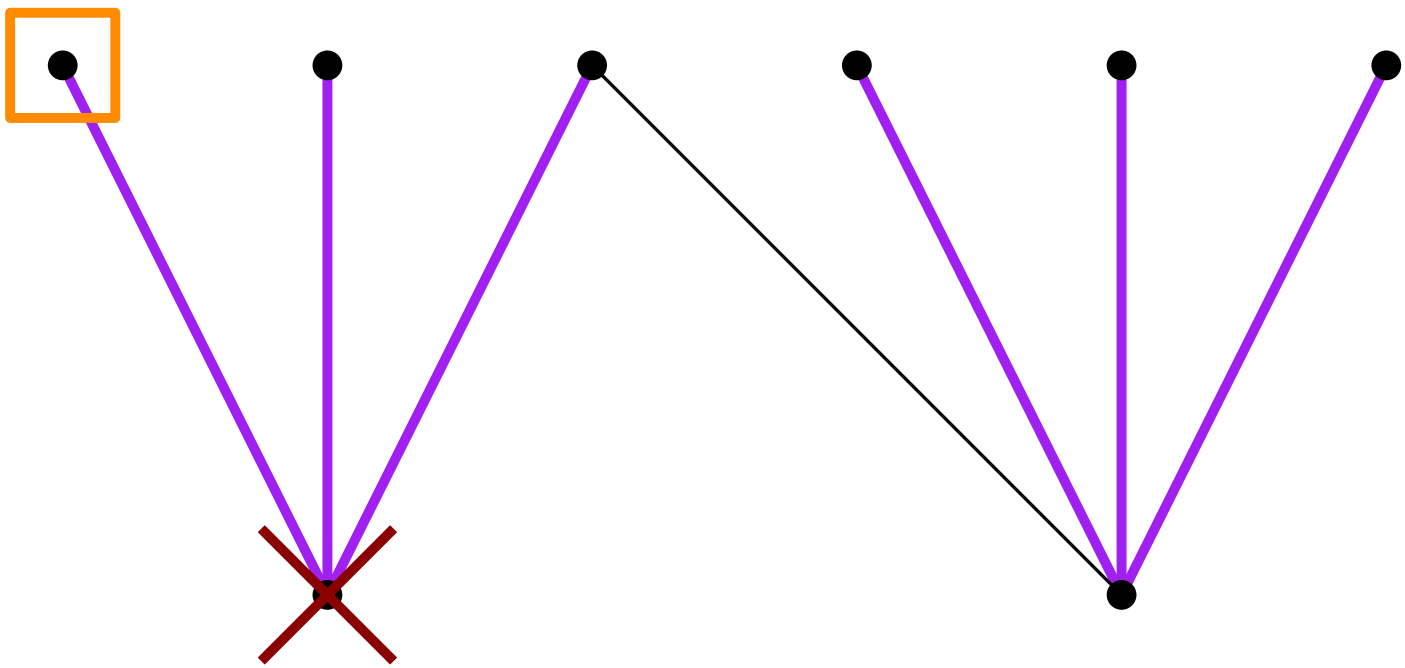

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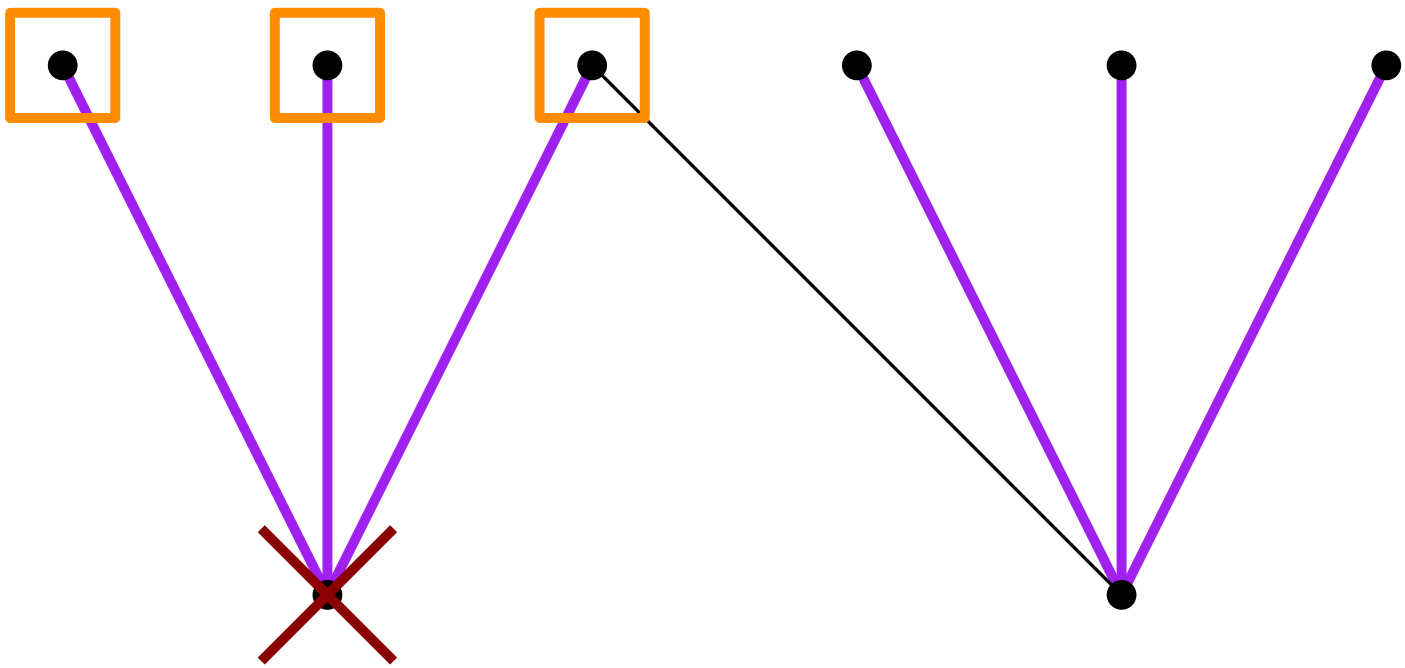

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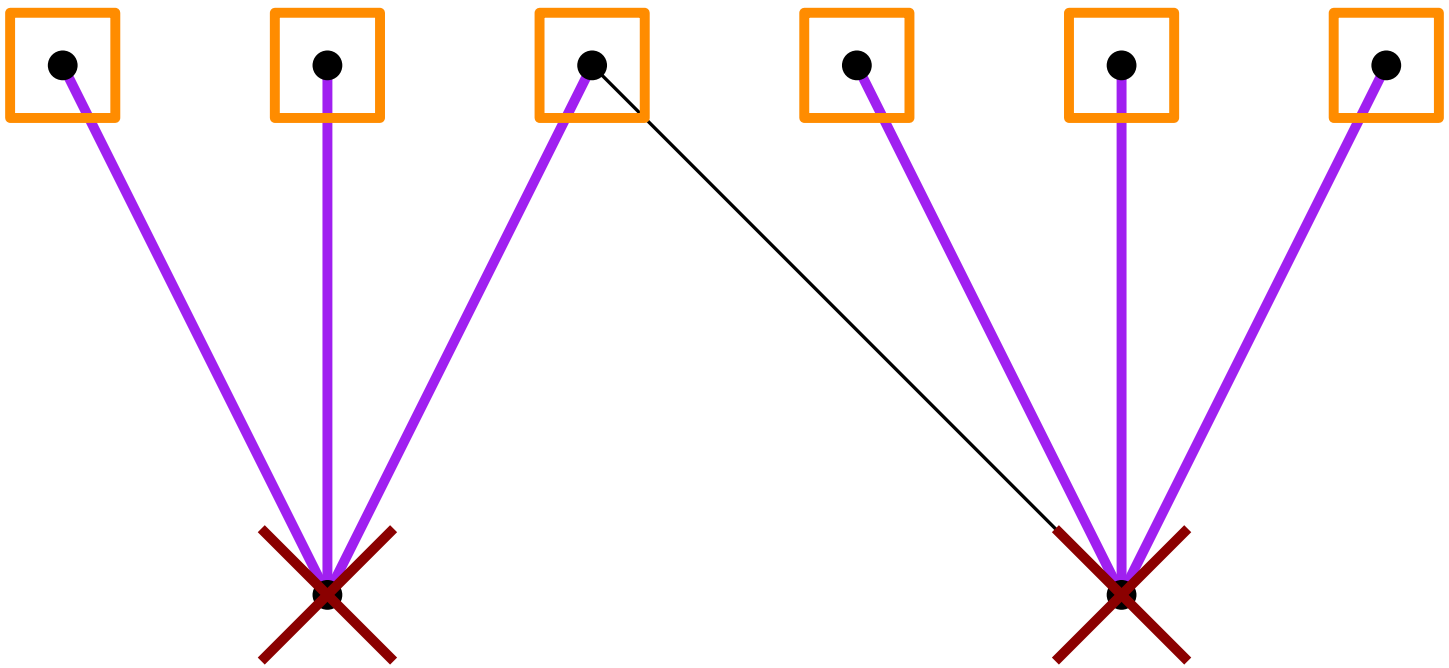

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Counter-example to the Normal Graph Conjecture!

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- **Maximum Weight Stable Set in (P_7, bull)-free graphs and ($S_{1,2,3}$, bull)-free graphs**.
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- **Colouring squares of claw-free graphs**.
  with R. de Joannis de Verclos and Ross J. Kang.
  Published in *Canadian Journal of Mathematics*.
- **Decomposition techniques applied to the Clique-Stable set Separation problem**.
  with N. Bousquet, A. Lagoutte and F. Maffray.
  Accepted in *Discrete Mathematics*.
- **Polynomial Cases for the Vertex Coloring Problem**.
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