Discussions about High-Quality Embedding on Quantum Annealers

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Abstract. Quantum Annealers (QA) such as D-Wave systems constitute a noisy implementation of the Adiabatic Quantum Computing process (AQC) and are used to find the ground state of an Ising problem. Qubit interconnections of a quantum chip are usually limited, and finding a good mapping of the Ising problem onto the quantum chip can be challenging. In fact, even defining what is a *high-quality embedding* is not trivial. After presenting a short review of existing embedding methods, we propose different experiments that could identify important criteria to consider while mapping problems on Quantum Annealers.

1 Introduction

AQC applied to optimization problems is a computational process introduced in [5] for optimization perspectives. This process describes the state evolution of conservative systems with a linear interpolation of two time-independent Hamiltonians H_M and H_C:

$$H(t)=\left(1-\frac{t}{T}\right)H_M+\frac{t}{T}H_C \tag{1}$$

It is seen that when $t=0, H(t)=H_M$ and when $t=T$ where T is the ideal annealing time, $H(t)=H_C$. The adiabatic theorem tells that if a quantum state is prepared into the initial ground state $|\psi_0\rangle$ of the Hamiltonian H_M and if t varies slowly enough from 0 to T, the quantum state $|\psi_t\rangle$ will stay close to a ground state of $H(t)$. The initial state $|\psi_0\rangle$ has to be easily prepared and must be a ground state of the Hamiltonian H_M. Additionally, H_M and H_C must not commute.
Quantum annealers, such as D-Wave systems [1], implement a noisy adiabatic evolution and are designed to minimize the Ising cost function $H_C(s)$ taking an input vector $s=(s_1,s_2,...,s_n)$ with $s_i\in\{+1,-1\}$ where h_i and J_{ij} represent qubits auto-coupling and coupling strength:

$$H_C(s)=-\sum_{i=0}^{n}h_i s_i-\sum_{i,j=1}^{n}J_{ij}s_i s_j \tag{2}$$

The topology of D-Wave quantum annealers is sparse. Hence, An efficient method is required to find an adequate mapping of qubits to embed problems on these chips (limiting the number of variable duplications).

2 Embedding Ising Problems on D-Wave Systems

Consider a source graph $G_s = (V_s, E_s)$, which models an Ising problem to be mapped onto a target graph $G_t = (V_t, E_t)$, which models the target quantum chip. The problem of mapping an Ising problem onto a quantum chip can be defined as follow :

Given a source graph $G_s = (V_s, E_s)$ and a target graph $G_t = (V_t, E_t)$, the goal is to find a mapping function : $\phi : V_s \rightarrow V_t \times V_t$ such that :
1. *each vertex $v \in V_s$ is mapped onto a connected subgraph $\phi(v)$ of V_s.*
2. *each connected subgraph must be vertex disjoint $\phi(v) \cap \phi(v') = \emptyset$, with $v \neq v'$.*
3. *each edge $e \in E_s$ is mapped onto at least one edge in E_t : $\forall(u,v) \in E_s, \exists u' \in \phi(u), \exists v' \in \phi(v)$, such that $(u',v') \in E_t$.*

Considering quantum annealers such as D-Wave systems, the graph G_t is very sparse and strongly limits the size and density of the source graph G_s that can be embedded into these chips. Polynomial algorithms used to decide if G_s can be embedded into G_t exist but do not report on the mapping function. In the theory of graph minors, Robertson and Seymour [10] have shown that for fixed G_s, there exists a polynomial algorithm to find its embedding on G_t. However, G_s is not fixed in our case, and the existing algorithm still has an exponential running time in the size of G_s.

3 Previous Work

Several attempts have been made to design efficient methods to find mappings of Ising problems on QA. These attempts can be divided into two categories.
The first approach is to look for the embedding of complete graphs with near-optimal embedding, considering the structure of the target graph. The first work was proposed by V. Choi [3], which provides an optimal embedding of complete graphs on triangular layouts (TRIAD scheme). This preliminary work was completed by C. Klymko et al. [6], who proposed a minor embedding method tailored to find clique embedding on lattices composed of regularly dispatched fully connected bipartite subgraphs. This method considers inoperable qubits (the target graph usually contains a few disabled qubits) and generates valid embeddings derived from the initial near-optimal clique embedding.
The second approach considers embedding algorithms of unknown structured input graphs on partially-known or unknown target graphs. An initial and generic heuristic was presented in [2] and is implemented in [4]. This algorithm is composed of two steps: the first one consists in finding an initial mapping for each logical qubit allowing overlapping (i.e., a vertex $v \in V_t$ may map more than one vertex $\phi(v)$ in V_s). The second step is a refinement where the mapping is iteratively improved by removing a vertex mapping $\phi(v)$ and looking for a better mapping for this vertex, minimizing the overall number of physical vertices. The quality of the mapping of a vertex is computed with a cost function. An output graph without any overlapping is considered valid. The refinement phase ends when no improvements have been made during a specific number of tries. Several other heuristics have been reusing this algorithm with the

addition of pre-processing phases as for Layout-Aware Minor Embedding [8,9], Spring-based MinorMiner (SPMM) and Clique-Based MinorMiner (CLMM) [14]. Another heuristic named Probabilistic-Swap-Shift-Annealing (PSSA) based on the simulated annealing algorithm was proposed in [12] and enhanced in [11]. This algorithm seems to be efficient, especially when the target graph G_{t} has a structure of a King's graph. All the previous methods build chains of logical qubits. However, chains usually break at their extremities when the chain strength is insufficient. A recent method proposed an embedding based on chains of cliques [7], which reduced chain break frequency and lowered the required couplings energy.

4 High-Quality Embedding

The characteristics defining a *high-quality embedding* are still poorly understood. Current embedding methods attempt to minimize the final number of physical qubits. However, several other factors may impact the quality of the embedding. We propose three questions and related experiments to understand better the important criteria to consider while embedding problems on quantum annealers.

What is the best structure for a logical qubit ? Evaluating the best possible structure for a logical qubit could guide the embedding heuristic. The error propagation on a logical chain usually starts at its boundaries [13]. For small chains, especially when the majority vote is crucial, replacing chains with cliques or cycles could be interesting, as in [7]. Error propagation could also be studied for logical qubit structures like trees. An experimental sampling of the same problem with different logical qubit structures, letting the coupling strength J_{ij} constant for each implementation, could help to identify such preferred structures.

Is there a maximum chain length that should not be exceeded ? Setting large negative values for each J_{ij} coupler inside a logical qubit theoretically maintains the problem structure. However, J_{ij} coupling strength is limited for D-Wave devices (e.g., $-1 < J_{ij} < 1$ for Advantage6.1 annealer) and is automatically rescaled when the problem is mapped to the quantum chip. Maintaining ferromagnetic coupling on large chains of physical qubits can be hard without setting large chain strengths. Experimental bounds considering the maximum length of the chain could be determined. This bound should depend on the initial distribution of h_i and J_{ij} weights and the precision of the quantum annealer.

Does the chain's distribution impact solution finding ? Studying the distribution of chains on the target topology (i.e., sparsity versus concentration; uniform chain length versus other chain length distributions) could give insight into the optimal allocation and duplication of variables. This study could be done by embedding the same problem with the same amount of qubit duplications, playing on their sparsity and chain length distributions.

These three experiments' results will help design the global objective function to maximize while searching for a *high-quality embedding*.

5 Conclusion

Evaluating the quality of an embedding is crucial to enhance the mapping of problems on quantum annealers. The embedding strongly impacts the ability to solve a problem that does not match the quantum chip topology. We propose several experiments that may identify criteria used to evaluate the quality of an embedding. This work is a first step to propose a quality-driven embedding method for quantum annealing-based processors.

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