Quantum Computing for solving the 3SAT problem by reduction to the MIS combinatorial optimization problem

Samuel Deleplanque*

*CNRS, Centrale Lille, JUNIA, Univ. Lille, Univ. Valenciennes, Lille Cedex, France

1 Introduction

This work tries to anticipate the main issues related to the D-Wave machines topology for solving the 3Sat problem. The polynomial reduction to the Maximum Independent Set Problem (MIS) is employed for solving a 3Sat instance to furnish a less dense graph to the machine. These analog quantum machines, based on the quantum annealing process ([3]), takes a Quadratic Unconstrained Binary Optimization model (QUBO) as input, which is a specific mathematical program easy to build considering the MIS.

2 Topologies, Embedding and Chains Breaks

Quantum computing will have a significant impact on the Combinatorial Optimization (CO) domain if the universal gate-based quantum machines integrate qubits with a lower error rate than NISQ. Today, to solve CO problems, we obtain more credible results with analog machines such as Pasqal or D-Wave quantum computers which are based on transverse Ising models which are isomorph to the QUBO models. The later type of machines has up to 5k qubits used in an adiabatic process named quantum annealing. The main difficulty is the mapping process (embedding) from the QUBO (or Ising) model to the qubit graph. Indeed the topology is here static (contrary to the Pasqal machines which can dynamically calculate a topology based on a given QUBO) and the mapping process is another optimization problem which can be very complicated to solve even if the instance is small. Figure 1 shows the topologies of the most recent D-Wave machines where the connectivities are 6, 14, 20 from left to right, respectively.

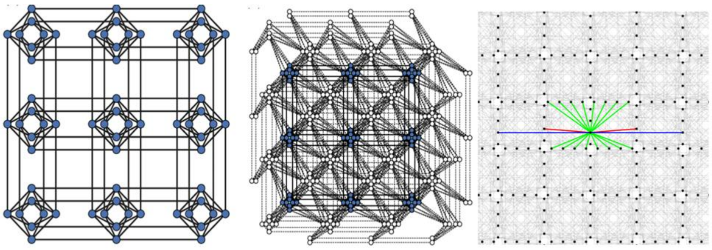

Figure 1: 3 D-Wave machines topologies. From left to right: Chimera, Pegasus and Zephir.

From a QUBO, the machine will apply a transformation to obtain a larger problem fitting with the qubits graph. Then, this transformation requires more qubits to create logical qubits which correspond, for example, to two physical qubits which must take the same value. However, the measure of these physical qubits can give different values; this is the *chain breaks* problem.

3 3Sat polynomial reduction to the MIS and QUBO model

This work considers the 3SAT $\leq_p$ MIS reduction with the aim of obtaining a less dense graph than the one we would have by relaxation on all 3Sat constraints $x_i + x_j + x_k > 0$ into the QUBO. Let's take an easy example to illustrate the transformation from a 3Sat instance to a MIS instance. Considering 3 variables and 3 clauses, we try to solve the following instance:

$$C_1: (x_1 \vee x_2 \vee \overline{x_3})$$
$$C_2: (\overline{x_1} \vee x_2 \vee x_3)$$
$$C_3: (x_1 \vee \overline{x_2} \vee x_3)$$

From the 3 variables we create 9 new variables, one per element of each clause. Taking each variable as a node in a graph, we consider a 3-clique per clause, and we also connect nodes if they correspond to the same initial variables and if one is the complement of the other. Finally, we obtain the graph of the Figure 2.

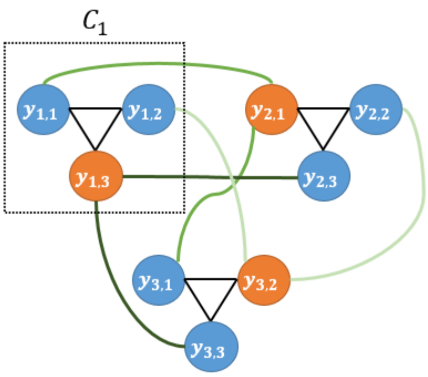

Figure 2: Graph representation of the MIS obtained from the 3Sat instance.

After reconstruction, we obtain the following solution:

$$x_1 = y_{1,1} = 1$$
$$x_2 = y_{3,2} = 1$$
$$x_3 = \overline{y_{2,3}} = 0$$

According to this transformation scheme, we generalize the process by noting y_{ij} a binary variable such that $y_{ij} = 1$ if y_{ij} is in the maximum independent set and 0 otherwise, with i = 1..|C| and j = 1..|X|, and with C the set of clauses, X the set of nodes, E the edges, in a graph G such that G = (X, E). Since we want to maximize the number of nodes (*true* variables) to be selected in the set while two connected nodes cannot be both selected, the related linear and constrained binary program is:

$$\text{Max} \sum_i \sum_j y_{ij}$$
$$\text{s.t.:}$$
$$y_{ij} + y_{kl} \leq 1 \qquad \forall (i,j),(k,l) \in E$$
$$y_{ij} \in \{0;1\}$$

According to a well-known QUBO transformation rule (see [2]), the constraints $y_{ij} + y_{kl} \leq 1$ are equivalent to the quadratic constraints $y_{ij}y_{kl} = 0$. Hence, with λ a multiplier, we obtain the following QUBO for the Maximum Independent Set Problem:

$$Min\ (-\sum_i \sum_j y_{ij} + \lambda \sum_{(y_{ij},y_{kl}) \in E} y_{ij}y_{kl})$$

The general scheme of resolution is given by the following diagram:

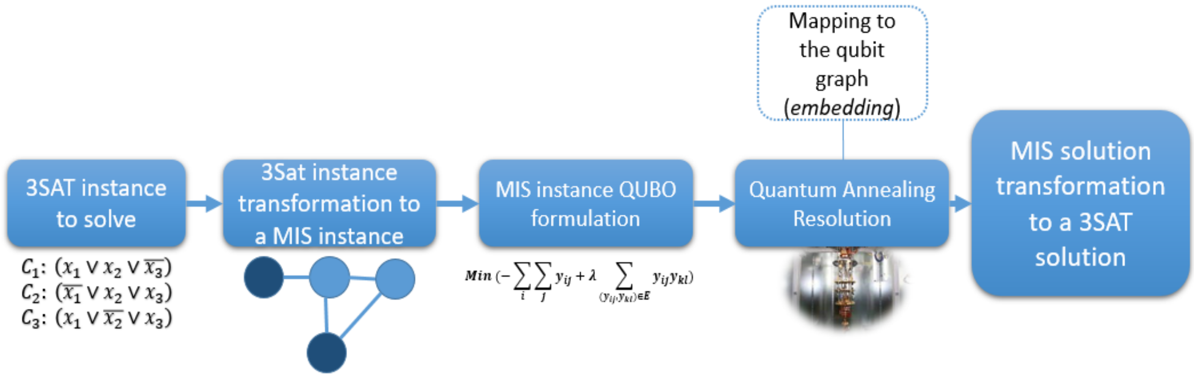

Figure 3: General resolution scheme.

4 Results

Table 1: Results on a 4 variables 11 Clauses instance according to the Annealing Time et the number of Anneals. Machine: Advantage 6.1 (Pegasus topology)

Annealing Time (μs)	Anneals Number	Chain Breaks Rate	Opt. Sol. Number
0.5	7000	High	92
1	7000	Average	6
10	7000	Average	2
100	2500	Average	3
1000	500	Average	1
2000	250	High	0

Other experiments had been done using different topologies showing the importance of a good topology. For a 3Sat instance with 4 variables and 6 clauses, resulting to a MIS instance with 18 variables, the Chimera topology requires 61 qubits while the Pegasus topology needs 31.

5 Preliminary Conclusions

Clearly the embedding process is a drag on a good resolution process. In this work, the 3SAT $\leq_p$ MIS reduction a been used to obtain a less dense graph in a view of a lighter connectivity (degree) of an instance. However the connectivity of the qubit graph topology is not the only one problem that makes the embedding process time-consuming. Other experiments must be made, especially on a larger set on instances to conclude.

References

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[2] Glover, F., Kochenberger, G., & Du, Y. (2019). Quantum Bridge Analytics I: a tutorial on formulating and using QUBO models. 4OR, 17, 335-371.

[3] Kadowaki, T., & Nishimori, H. (1998). Quantum annealing in the transverse Ising model. *Physical Review E*, *58*(5), 5355.

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